3.7 Stability of Equilibrium Points **83**

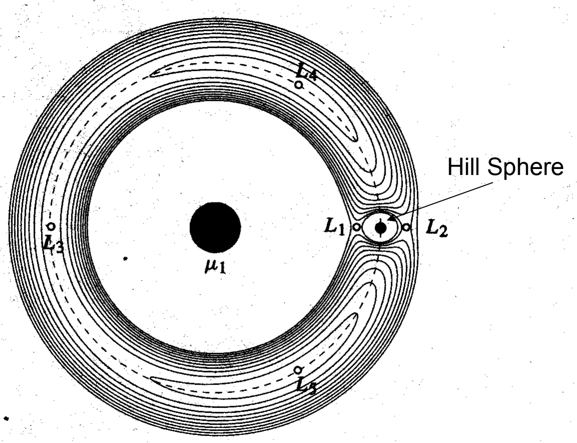
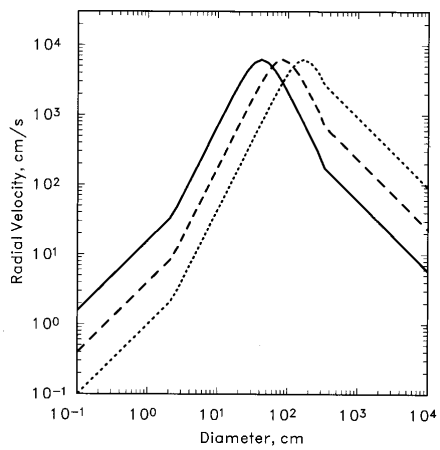


Fig. 3.9. The location of the Lagrangian equilibrium points (open circles) and associated zero-velocity curves for a mass $\mu_2 = 0.01$. The dashed line denotes the circle of unit radius centred on the mass μ_1.

FIGURE 12.11 The inward radial drift rates of solid particles in a protoplanetary disk as a function of size for three values of density: 0.5 (solid line), 2.0 (dashed line), and 7.9 (dotted line) g cm^{-3}. Gas parameters are the same as for Figure 12.10. Small particles, with small mass/surface area ratios, are strongly coupled to the gas and compelled to move with (nearly) its angular velocity. As this is less than the keplerian orbital rate, they feel a residual component of the Sun's gravity, and settle inward at a terminal velocity at which gas drag balances this radial acceleration. Thus, larger and/or denser particles drift more rapidly in this regime. Bodies with large mass/surface area ratios travel in (nearly) keplerian orbits, moving faster than the gas. They experience a 'headwind' that causes their orbits to decay; larger and/or denser bodies are less affected by this drag, so the decay rate decreases with increasing particle radius. The radial velocity reaches a peak at the transition between these regimes, at sizes of about a meter. The abrupt changes in slope result from transitions between drag laws for different Knudsen and Reynolds numbers. (Courtesy: Stuart J. Weidenschilling)



Growth from planetesimals to planetary embryos:

For bodies > 1 km major forces are gravitational interaction and physical collisions and gas drag.

The larger the body the more quickly it grows (runaway accretion).

Collision between planetesimals:
v speed at large distances
v_e escape speed
Impact velocity $v_i \geq v_e$
$v_i \geq 6$ m s^{-1} for rocky 10 km body.
Restitution velocity = v_i ε with ε ≤ 1. If $v_i\,\varepsilon \leq v_e$ particle accretes sooner or later.
This is the reason for runaway accretion.
Small grains do not accrete on large grains because of too high relative speed v.
Sandblasting of growing planetesimals.

$$v_i = \sqrt{v^2 + v_e^2}, \quad v_e = \left(\frac{2G(m_1+m_2)}{R_1+R_2}\right)^{1/2}.$$

Growth time of planets:

Mass accretion, ρ_s volume density of planetesimal swarm:

$$\frac{dM}{dt} = \rho_s v \pi R^2 \mathcal{F}_g, \quad \mathcal{F}_g = 1 + (v_e/v)^2.$$

Transfer to surface mass density of planetesimals σ_ρ (g cm^{-2}):
n is mean angular motion in orbit

$$H_z = \frac{v}{\sqrt{3}n}.$$

$$\sigma_\rho = \sqrt{\pi}\,\rho_s H_z.$$

ρ_p density of planetary embryo

$$\frac{dR}{dt} = \sqrt{\frac{3}{\pi}}\frac{\sigma_\rho n}{4\rho_p}\mathcal{F}_g,$$

For Earth $F_g = 7$, $\sigma_\rho = 10$ g cm^{-2}, n = 2x10^{-7} s^{-1}, $\rho_p = 4.5$ g cm^{-3}, growth time 2x10^7 yr, or better 10^8 yr, if depletion of planetesimals in later stage of accretion is considered.

Problems with outer planets. For Jupiter $\sigma_\rho = 3$ g cm^{-2}, heavy element mass 15-20 Earth masses, growth time > 10^8 yr. Surface density of solar nebula drops ~ r$^{-3/2}$, growth time of Neptune is many times the solar system age.
Problems to make outer planets in time.
Further problem: gas accretion of outer planets and Jupiter.

Runaway growth of planetary embryos

If $v \geq$ or $\approx v_e$ and after impact (sticking factor) $\leq v_e$,
then growth ~ R^2 (R radius of planetesimal).

If $v \ll v_e$ then growth ~ R^4.
F_g gravitational enhancement factor can exceed 1000, but gravitational stirring prevents F_g from getting much larger than this. At this stage the growth is slowed down, other growing embryos can catch up in size (oligarchic growth).

Area within reach of the growing embryo is ~4 times its Hill sphere.
Hill sphere: sphere of gravitational influence (limited by Lagrange points, see next view graph).
Radius R_H of Hill sphere:

$$R_H = \left(\frac{m_2}{3(m_1 + m_2)} \right)^{1/3} a,$$

Mass of planetary embryo which has accreted all mass within a ring of width $2\Delta r_\odot$:

$$M = \int_{r_\odot - \Delta r_\odot}^{r_\odot + \Delta r_\odot} 2\pi r' \sigma_\rho(r') dr' \approx 4\pi r_\odot \Delta r_\odot \sigma_\rho(r_\odot).$$

If $\Delta r_\odot = 4\ R_H$ we obtain maximum mass M_i (in g) to be accreted by a planetary embryo orbiting a star of 1 Mo:

$$M_i \approx 1.6 \times 10^{25} (r_{AU}^2 \sigma_\rho)^{3/2},$$

For Earth $M_i = 5\ 10^{26}$ g. 1 Earth mass = $6\ 10^{27}$ g.

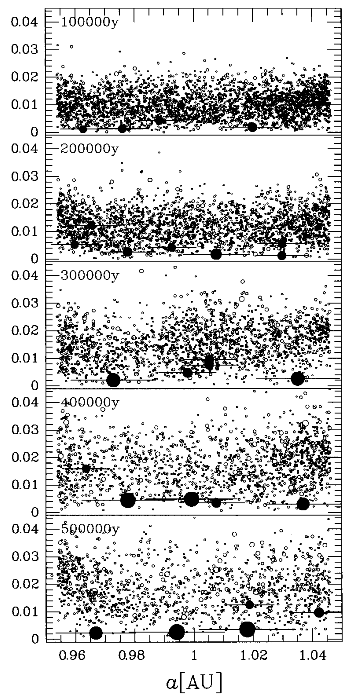
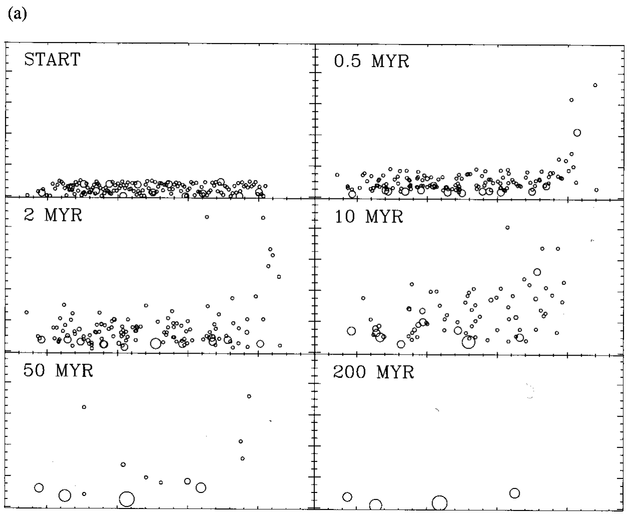


FIGURE 12.12 Snapshots of a planetesimal system on the *a*–*e* plane. The circles represent planetesimals and their radii are proportional to the radii of planetesimals. The system initially consists of 4000 planetesimals whose total mass is 1.3×10^{27} g. The initial mass distribution is a power with index $\zeta = -2.5$ over the mass range 2×10^{23} g $\leq m \leq 4 \times 10^{24}$ g. The system is followed using an *N*-body integrator, and physical collisions are assumed to always result in accretion. The numbers of planetesimals are 2712 (t = 100 000 yr), 2200 (t = 200 000 yr), 1784 (t = 300 000 yr), 1488 (t = 400 000 yr), and 1257 (t = 500 000 yr). The filled circles represent planetary embryos with mass larger than 2×10^{25} g, and lines from the center of each planetary embryo extend 5 R_H outwards and 5 R_H inwards. (Kokubo and Ida 1999)

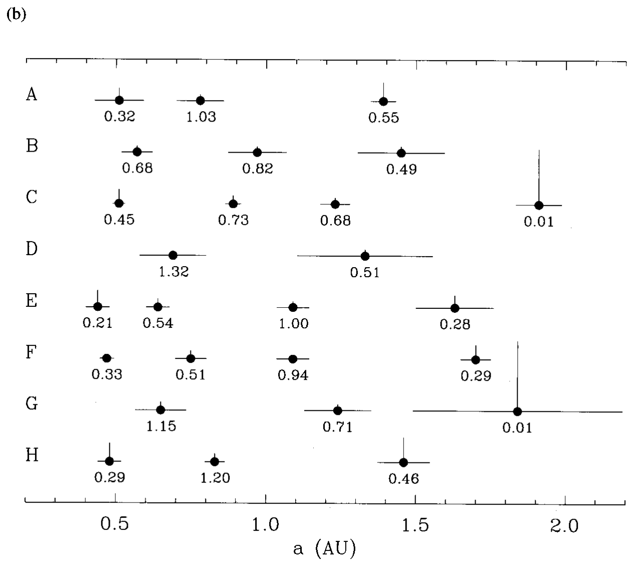
Making planetary embryos close to the Earth:
In the terrestrial planet region, to complete terrestrial planets, further accretion among protoplanets (giant impacts) is necessary. The Earth's moon may have formed by such an impact. Collisions may be induced by perturbations by giant planets or by the embryos themselves.

see Eiichiro Kokubo, Planetary accretion: From Planetesimals to Protoplanets, Rev. Mod. Astronomy 14, 117-132, 2001.

FIGURE 12.13 (a) Simulation of the final stages of terrestrial planet growth in our Solar System using an *N*-body code that includes Jupiter and Saturn and that assumes all physical collisions lead to mergers. Planetary embryos are represented as circles whose radii are proportional to the embryos' radii. The locations of the planetary embryos are displayed in semimajor axis–eccentricity phase space at the times indicated.



I. (b) Synthetic terrestrial planet systems produced by eight different *N*-body simulations of the final stages of planetary accretion. The final planets are indicated by filled circles centered at the planet's semimajor axis. The horizontal line through each circle extends from the planet's perihelion to its aphelion; the length of the vertical line extending upward from a planet's center is proportional to its inclination. The numbers under each circle represent the planet's final mass in $M_{\oplus}$. The results of the simulation shown in part (a) are presented in row E. (Courtesy: John Chambers)



Origin of solar systems: Organization

Lecture (KJ):

Introduction and overview
Dense molecular clouds, photo-dissociation regions and protostars

Protoplanetary disks
Equilibrium condensation of a solar nebula

Meteorites and the early solar system

Origin of giant planets

Comets and the early solar system

Student talks:

Origin of the elements and Standard Abundance Distribution

Agglomeration of planetesimals and protoplanets

Isotope chronology of meteorites and oxygen isotopes

Extrasolar planets

Transneptunian Objects

Lewis, "Physics and Chemistry of the Solar System"

Gas capture from the solar nebula

$n_s/n_\infty \sim p_s/p_\infty \sim \rho_s/\rho_\infty = \exp[\mu V^2_{esc}/2RT_\infty]$

s subscript: surface of planet

Why do we need solid planetesimals to form planets?

Escape speed v_{esc} plotted versus temperature away from the body for different temperature ratios surface/infinity relevant to terrestrial planets (isothermal case).

If we do not have a solid body at the center we cannot enhance gas pressure with respect to the surrounding pressure and cannot accrete gas effectively.

Lewis, “Physics and Chemistry of the Solar System”

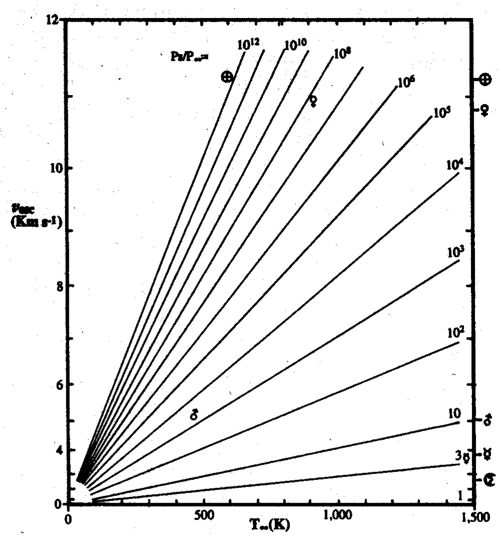


Figure IV.45 Isothermal gas capture from the solar nebula. Contours of surface gas pressures relative to the nearby unperturbed central plane nebular pressure are given.

Conclusion:

• Mercury and moon cannot capture any gas.
• Venus and earth could have captured masses of solar material comparable to the mass of the planet.
• If one takes into account the gravity of the accreted gas Earth and Venus might have become Jovian planets.
• However, as the planets move in the protoplanetary disk, interaction time with the planetary potential is limited in time and the planets cannot captue as much mass as they could in a static situation.

Table IV.8 Minimum Mass of the Primitive Solar Nebula

Planet	Mass (10^{26})	F^a	$M_{solar}(10^{26}g)$	$r_{ann}(10^{13}cm)$	$A_{ann}(10^{26}cm^2)$	$\sigma = M/A(g\ cm^{-2})$
Mercury	3.3	350	1,160	0.33–0.83	1.82	637
Venus	48.7	270	13,150	0.83–1.29	3.06	4300
Earth	59.8	235	14,950	1.29–1.89	6.00	2500
Mars	6.4	235	1,504	1.89–3.20	20.95	72
Asteroids	0.1	200	20	3.2–6.0	80.9	0.25
Jupiter	19,040	5	95,200	6.0–11.0	267	355
Saturn	5,695	8	55,560	11.0–21.5	1072	42.4
Uranus	870	15	13,050	21.5–36.8	2802	4.7
Neptune	1,032	20	20,640	36.8–52.0	4240	4.9
Pluto	0.1	70	7	52–70	6900	0.001

[a] *F* is the factor by which the planetary mass must be multiplied to adjust the observed material to solar composition.

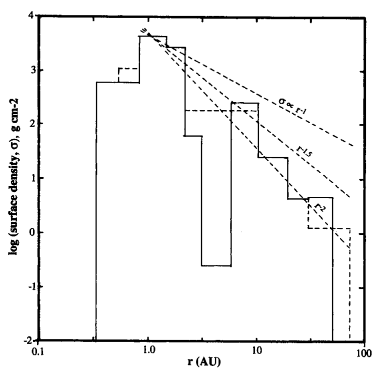


Figure IV.33 Mass distribution in the solar nebula. A mean slope of $r^{-1.5}$ to $r^{-2.0}$ is suggested. The inner and outer edges appear sharply truncated. The inner edge is certainly due to the infall of matter from that region into the forming Sun. The outer edge may be due to a finite scale size of the original nebular condensation at the time of its last Jeans instability.

Formation of the Giant Planets

Wuchterl G., Guillot, T., Lissauer, J.J., Protostars and Planets IV, 1081-1109.

The 4 giant planets contain 99.5% of the angular momentum and 0.13% of the mass of the solar system, but more than 99.5 of the mass of the planetary system. Macroscopic angular momentum transfer process occurs through turbulent viscosity.

The minimum reconstituted nebula mass is the total mass of solar composition material needed to provide the observed planetary/satellite masses and compositions by condensation and accumulation. It amounts to a few percent of the central body, both for the solar nebula and circum-planetary nebulae.

The total angular momenta of the satellite systems are only a few percent of those of the central body, however.

Even if giant planets had kept the angular momentum they got through Keplerean shear from the nebular disk, they still would not rotate critically.

I.e. when studying the formation of giant planets we may neglect rotation.

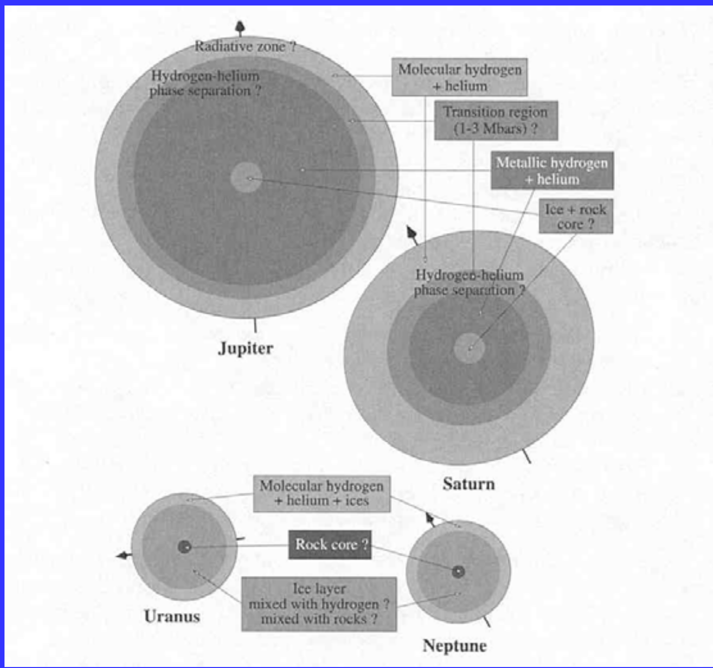
Interior of the giant planets:

Construction of interior models matching the observed gravitational fields.

With the exception of Uranus the giant planets emit more energy than received from the Sun. They are *hot* inside, and *convective*. These conclusions should also hold for Uranus.
Envelopes of all four giant planets should be homogeneously mixed, but there are caveats:

1. Condensation and chemical reactions alter chemical composition (these should be confined to the external regions).
2. A first-order phase transition (such as the one between molecular and metallic hydrogen) imposes an abundance discontinuity across itself.
3. Hydrogen-helium phase separation might occur and lead to a variation of the abundance of helium in the planet.
4. The envelopes of Uranus and Neptune are small and enriched in heavy elements; it is thus conceivable that molecular weight gradients inhibit convection and yield nonhomogeneous envelopes.

The interiors of Jupiter, Saturn, Uranus and Neptune, according to conventional wisdom



Uranus and Neptune:
Three layers: "Rock" core, "ice" layer (H_2O, CH_4, NH_3), and hydrogen-helium envelope.
Envelope enriched in heavier elements:
30x more carbon in the form of CH_4 in their tropospheres. H_2O may also be enriched but condenses out already in deeper layers.
Ice/rock ratio ≈ 10 or higher, but protosolar value ≈ 2.5. These nonhomogeneous regions probably date back to the accretion of these planets.

Jupiter and Saturn:
Simpler:
Core, inner envelope of metallic hydrogen, outer layer with hydrogen in the form of H_2.
Each layer is homogeneous, but He depleted in H_2 layer and therefore probably metallic layer enriched in He (and possibly Ne).
Models calculated by Giullot (1997, 1999) allow to infer the possible heavy element abundance in the metallic and molecular regions. Uncertainties caused by equation of state, interior temperature profile (convective, radiative) and rotation (solid, differential).

Note that Jupiter and Saturn may not have a core (judging from these models). But they need a core because otherwise, how could they form?

GIANT PLANET FORMATION 1087

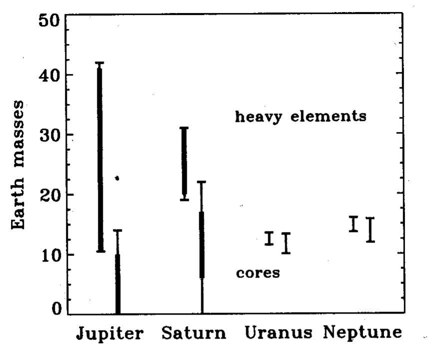


Figure 2. Limits on the abundances of heavy elements in the four jovian planets in our solar system. For each planet, the point on the left represents the total amount of high-*Z* material, whereas the (lower) point on the right shows the amount of heavy elements segregated into the planet's core. For Jupiter and Saturn, the thick lines represent solutions with additional constraints obtained from evolution models. Note the high level of uncertainty, especially regarding the core masses of Jupiter and Saturn. Models of Jupiter with small cores (i.e., less than 2 $M_\oplus$) require significant enrichments in heavy elements (i.e., more than 20 $M_\oplus$).

Gas Accumulation Theories:

Preplanetary disks are only weakly self-gravitating equilibrium structures, supported by centrifugal forces augmented by gas pressure. Any isolated, orbiting object below the Roche density is pulled apart by the stellar tides. Nebular densities are typically more than two orders of magnitude below the Roche density.

Compression is needed to confine a condensation of mass M inside its tidal or Hill radius $R_T = a\ (M/3Mo)^{1/3}$. A local enhancement of self-gravity is needed to overcome the counteracting gas pressure.

1. The *nucleated instability* model relies on the extra gravity field of a sufficiently large solid core (condensed material represents a gain of ten orders of magnitude in density, and therefore self-gravity, compared to the nebula gas).
2. A *disk instability* may operate on lengthscales between short-scale pressure support and long-scale tidal support.
3. An *external perturber* could compress an otherwise stable disk on its local dynamical timescales, e.g., by accretion of a clump onto the disk or rendezvous with a stellar companion.

Nebula Stability:

Preplanetary nebulae with minimum reconstituted mass are stable.

A moderate-mass nebula disk might be found that can develop a disk instablility leading to a strong density perturbation, especially when forced with a finite external perturbation. A density enhancement of a factor 100 can be obtained (Boss, see next slide). But the density enhancement at the surface of a 1 Earth mass core is between 10^5 and 10^7, for comparison (Lewis has 10^{12} at 1 AU).

Clumps forming as the result of such an instability (giant gaseous protoplanets GGPPs) are candidates to become proto-giant planets, but they must cool rapidly to stabilize (problems may arise because of high opacity) and they must form a core a posteriori.

Wuchterl has checked the stability of GGPPs.
- Alexander and Ferguson (1994) opacities.
- Time-dependent mixing length.
- Jeans-critical nebula of Jupiter's mass with T=10 K.
- Needs 1.8 x 10^4 yr to contract into tidal radius.
- Is fully convective from < 100 to 2 x 10^5 yr, when a radiative zone spreads out from the planet's center.

Boss, A.P.: "Gas Giant Protoplanet Formation: Disk Instability Models with Thermodynamics and Radiative Transfer", Ap. J. 563, 367-373, 2001.

Density enhancement of four orders of magnitude possible.

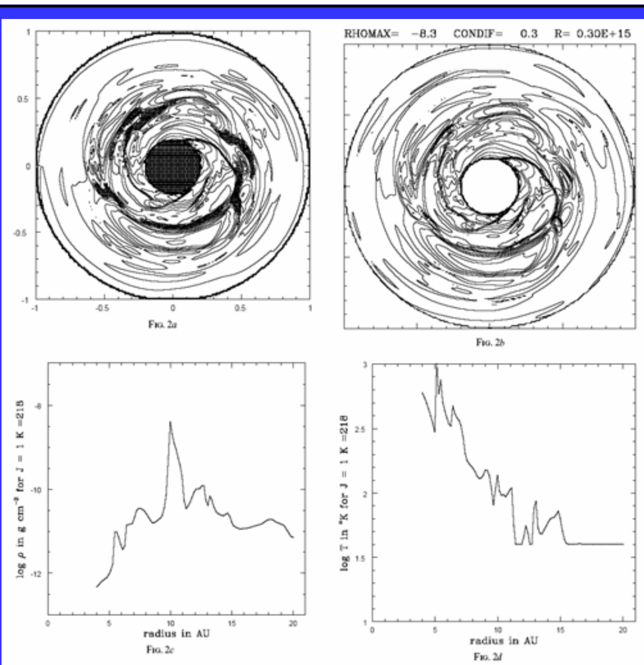


Fig. 2—(a, b) Equatorial density contours ($\rho_{max} = 5.0 \times 10^{-9}$ g cm^{-3}) and (c) radial density and (d) temperature profiles through the maximum clump density for model HR at 339 yr. Each plot extends to 20 AU in radius. Hatched regions in (a) denote densities above 10^{-10} g cm^{-3}. Contours denote changes by factors of 2 in density in (a) and (b). Figure 2b is similar to (a) except that the hatch marks are removed in order to show the contours in the high-density regions. The maximum density clump is located at 4 o'clock in (a) and (b) and at 10 AU in radius in (c) and (d).

Nucleated Instability:

Planetesimals (solids) in the solar nebula are small bodies surrounded by gas.

Idea of *critical core mass*: At a certain critical core mass the atmosphere could not be sustained, and isothermal, shock-free accretion (Bondy 1952) would set in.

Miniature stellar structure calculations with energy dissipation by impacting planetesimals replacing the nuclear reactions as an energy source.

Safronov and Ruskol (1982):
The rate of gas accretion is determined not by the rate of delivery of mass to the planet [like in Bondy accretion] *but by the energy losses from the contracting envelope.*

Simplified models:

Stevenson, D.J. 1982: Formation of Giant Planets. Planet. Sp. Sci. 30, 755-764.

• generalized opacity law $\kappa = \kappa_0 P^a T^b$
• core mass accretion rate M^{dot}_{core}, core density ρ_{core} inside tidal radius R_T
• "radiative zero solution" for spherical protoplanets with static, fully radiative envelopes, in hydrostatic and thermal equilibrium

The critical mass, defined as the largest mass to which a core can grow while forced to retain a static envelope, is given by

$$M_{\rm core}^{\rm crit} = \left[\frac{3^3}{4^4}\left(\frac{\mathcal{R}}{\mu}\right)^4 \frac{1}{4\pi G}\,\frac{4-b}{1+a}\,\frac{3\kappa_0}{\pi\sigma}\left(\frac{4\pi}{3}\rho_{\rm core}\right)^{1/3}\frac{\dot{M}_{\rm core}}{\ln\left(R_T/r_{\rm core}\right)}\right]^{3/7}$$

where $M_{\rm core}^{\rm crit}/M_{\rm tot}^{\rm crit} = \frac{3}{4}$ and R, G, and σ denote the gas constant, the gravitational constant, and the Stefan-Boltzmann constant, respectively.

Note that this model does not depend on nebular density or temperature, but strongly on the molecular weight μ ("superganymedean puffballs enriched in heavy elements").
Variation of a factor of 100 in M^{dot}_{core} leads to only to a 2.6 variation in the critical core mass. Model similar to proto-giants and leads to oscillation-driven mass loss.

Wuchterl, G. 1993: Icarus 106, 323-334.
Static solutions for protoplanets with convective outer envelopes, which occur at somewhat larger midplane densities than in minimum mass nebulae.

For a given core envelopes are larger and the critical core mass is reduced.

$$M_{\text{core}}^{\text{crit}} = \frac{1}{\sqrt{4\pi}} \frac{\sqrt{\Gamma_1 - \frac{4}{3}}}{(\Gamma_1 - 1)^2} \left(\frac{\Gamma_1}{G} \frac{\mathcal{R}}{\mu} \right)^{3/2} T_{\text{Neb}}^{3/2} \rho_{\text{Neb}}^{-1/2}$$

Γ_1 is constant first adiabatic exponent and $M_{\text{core}}^{\text{crit}}/M_{\text{tot}}^{\text{crit}}$ = 2/3.
In this case, the critical mass depends on the nebula gas properties and therefore on the location in the nebula, but is independent on core accretion rate.

Early phases of giant planet formation are dominated by the growth of the core. Envelopes remain close to static.
The nucleated instability was assumed to set in at the critical mass, originally as a hydrodynamic instability analogous to the Jeans instability. With the recognition that energy losses from the proto-giant planet envelopes control the further accretion of gas, it followed that quasi-hydrostatic contraction of the envelopes would play a key role.

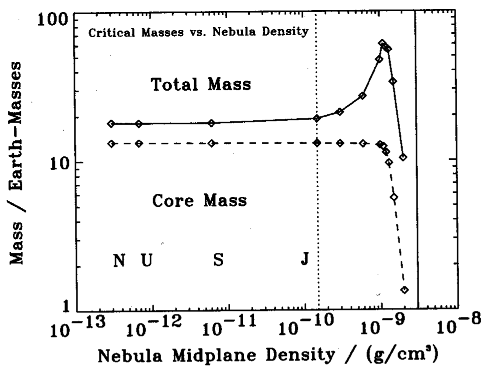


Figure 3. Critical masses of static protoplanets as a function of nebula midplane density. Critical total mass and core mass values are connected by a solid and a dashed curve, respectively. Observe the increased envelope masses and decreased core masses for the convective outer envelopes occurring at larger nebula densities. The conditions in the nebula correspond to Mizuno's minimum-mass nebula (Mizuno 1980); densities at the Neptune, Uranus, Saturn, and Jupiter positions are labeled by N, U, S, and J, respectively. They illustrate the constancy of the critical mass in the case of radiative outer envelopes. Densities to the right of the dotted vertical line are arbitrarily enhanced relative to the minimum-mass values, so that the outer envelopes become convective (see text). The solid vertical line gives an estimate for the critical density of a Jupiter-mass nebula fragment at Jupiter's position. The value plotted is the mean density of a condensation that is Jeans critical *and* fits into its Hill sphere.

Quasi-hydrostatic Models with Detailed Core Accretion:

Pollack et al. 1996, Icarus 124, 62-85.

very detailed in many respects (core accretion rate, planetesimal dissolution in envelope, treatment of energy loss via radiation and convection, equation of state), but:

1. The planet is assumed to be spherically symmetric.
2. Hydrodynamic effects are not considered in the evolution of the envelope.
3. The opacity in the outer envelope is determined by a solar mixture of small grains in most of the simulations. Solar abundances are also used to calculate the opacity in deeper regions of the envelope, where molecular opacities dominate.
4. The equation of state for the envelope is that for a solar mixture of elements.
5. During the entire period of growth of a giant planet, it is assumed to be the sole dominant mass in the region of its feeding zone, i.e., there are no competing embryos, and planetesimal sizes and random velocities remain small. A corollary of this assumption is that accretion can be described as a quasicontinuous process, as opposed to a discontinuous one involving the occasional accretion of a massive planetesimal.
6. Planetesimals are assumed to be well-mixed within the planet's feeding zone, which grows as the planet's mass increases, but planetesimals are not allowed to migrate into or out of the planet's feeding zone as a consequence of their own motion. Tidal interaction between the protoplanet and the disk, or migration of the protoplanet (see the chapters by Lubow and Artymowicz, Ward and Hahn, and Lin et al. in this volume), are not considered.

TABLE I
Properties of Planetesimals

	Component[a]			
Property	H_2O Ice	Rock	CHON	Total
mass fraction	0.397	0.308	0.295	1
density (g/cm^3)	0.92	3.45	1.5	1.39
latent heat[b] (erg/g)	[c]2.8×10^{10}	[c]8.08×10^{10}	[d] -7.0×10^{10}	1.54×10^{10}
vaporization temperature (K)	165	1500	650	-

[a] The three major components of the planetesimals are water ice, ferromagnesium silicates ("rock"), and organics ("CHON").
[b] The latent heats of ice and rock are endothermic, whereas that of the CHON is exothermic.
[c] Podolak *et al.* (1988).
[d] Estimated.

TABLE II
Key Model Parameters and Their Nominal Values

Parameter	Nominal Value
orbital distance	5.2 A. U.
planetesimal radius	100 km
other planetesimal properties	see Table I
initial planetesimal surface density	10 g/cm^2
fate of dissolved planetesimal	sinks to core interface
nebula temperature	150 K
nebula density	5.0×10^{-11} g/cm^3

TABLE III
Input Parameters

case	$\sigma_{init,Z}$ (g/cm^2)	$\sigma_{init,XY}$ (g/cm^2)	r_p (km)	a (A.U.)	T_{neb} (K)	ρ_{neb} (g/cm^3)	δ_s
J1	10.	700.	100	5.203	150	5.0×10^{-11}	1

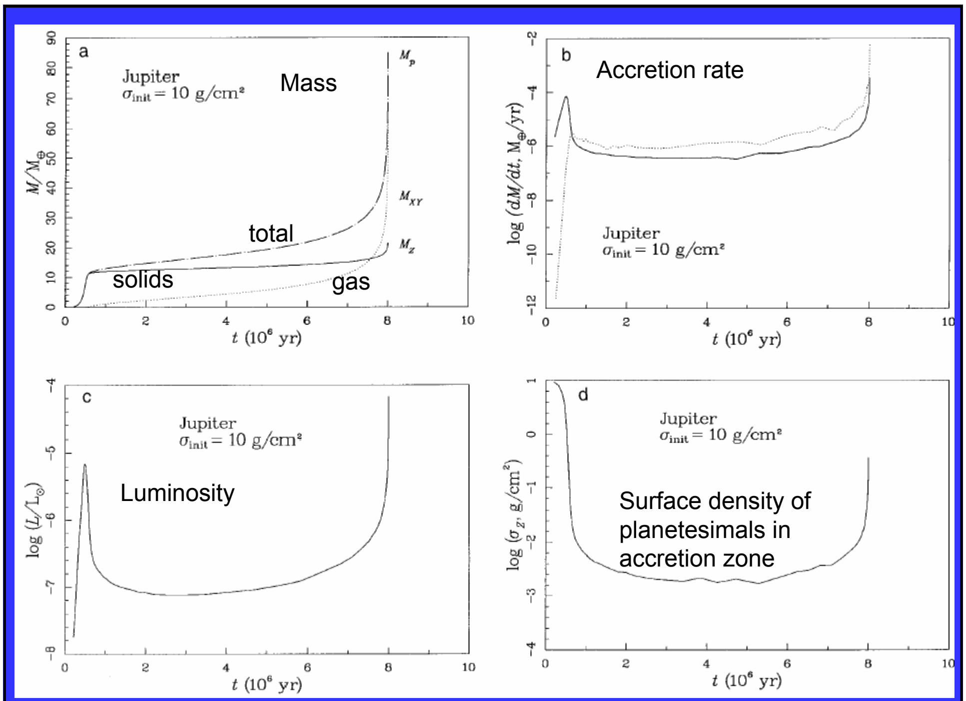


FIG. 1. (a) Planet's mass as a function of time for our baseline model, case J1. In this case, the planet is located at 5.2 AU, the initial surface density of the protoplanetary disk is 10 g/cm^2, and planetesimals that dissolve during their journey through the planet's envelope are allowed to sink to the planet's core; other parameters are listed in Table III. The solid line represents accumulated solid mass, the dotted line accumulated gas mass, and the dot–dashed line the planet's total mass. The planet's growth occurs in three fairly well-defined stages: During the first $\sim 5 \times 10^5$ years, the planet accumulates solids by rapid runaway accretion; this "phase 1" ends when the planet has severely depleted its feeding zone of planetesimals. The accretion rates of gas and solids are nearly constant with $\dot{M}_{XY} \approx 2\text{–}3\dot{M}_Z$ during most of the $\sim 7 \times 10^6$ years' duration of phase 2. The planet's growth accelerates toward the end of phase 2, and runaway accumulation of gas (and, to a lesser extent, solids) characterizes phase 3. The simulation is stopped when accretion becomes so rapid that our model breaks down. The endpoint is thus an artifact of our technique and should not be interpreted as an estimate of the planet's final mass. (b) Logarithm of the mass accretion rates of planetesimals (solid line) and gas (dotted line) for case J1. Note that the initial accretion rate of gas is extremely slow, but that its value increases rapidly during phase 1 and early phase 2. The small-scale structure which is particularly prominent during phase 2 is an artifact produced by our method of computation of the added gas mass from the solar nebula. (c) Luminosity of the protoplanet as a function of time for case J1. Note the strong correlation between luminosity and accretion rate (cf. b). (d) Surface density of planetesimals in the feeding zone as a function of time for case J1. Planetesimals become substantially depleted within the planet's accretion zone during the latter part of phase 1, and the local surface density of planetesimals remains small throughout phase 2. (e) Four measures of the radius of the growing planetary embryo in case J1. The solid curve shows the radius of the

Hydrodynamic Accretion beyond the Critical Mass

Wuchterl's models are nonlinear, convective, radiation hydrodynamical calculations of core-envelope proto-giant planets that follow the evolution without *a priori* assuming hydrostatic equilibrium and which *determine* whether envelopes are hydrostatic, pulsate or collapse and at what rates mass flows onto the planet.

- Spherical symmetry
- Core accretion rate assumed to be either constant or according to the particle in box approximation (see e.g. Lissauer 1993).
- Other assumptions of quasihydrostatic models hold here also.

First calculation: Pulsation driven wind. After a large fraction of the envelope mass has been pushed back into the nebula, the dynamical activity fades, and a new quasi-equilibrium state is found that resembles Uranus or Neptune in core and envelope mass.

When can accretion occur?

Pulsations and mass loss do not occur when "no dust", zero metallicity opacities are used.
Static critical core mass =1.5-3 M_{Earth} for accretion rates 10^{-8} to 10^{-6} M_{Earth} yr^{-1}
Envelope accretion becomes independent of core accretion at ≈15 M_{Earth}.
Mach number = 0.01 at ≈ 50 M_{Earth}.
At a total mass of about 100 M_{Earth} the nebula gas influx approaches the Bondi rate, at 300 M_{Earth} the envelope collapses overall.

Even with realistic opacities there exist models leading to accretion.
E.g. at a nebula density of 10^{-9} g cm^{-3} (greater by a factor of 6.7 than Mizuno's (1980) minimum reconstituted mass nebula value) pulsations were damped and rapid accretion of gas set in and proceeded to 300 M_{Earth}. The spreading of convection into the outer envelope had damped the oscillations.

Future: Improved Convective Energy Transfer and Opacities

Formation of Extrasolar Planets (or any Giant Planet):

Hydrostatic models for *in situ* formation:

For giant planets to form close to the parent star high surface mass density of solids is required (because the larger Kepler shear near the star decreases the solid core's isolation mass unless the amount of solids is large.

The planet orbiting 2.1 AU from 47 UMa can form in ~2 Myr for σ = 90 g cm^{-2} but requires ~18 Myr for σ = 50 g cm-2.

The surface mass density of solids required to form giant planets at 0.23 AU (ρCrB) and 0.05 AU (51Peg) is prohibitively large unless orbital decay of planetesimals is incorporated into the models.
Ad-hoc assumption: *constant rate of solid body accretion.*
Model results for 51 Peg indicate: if the growth rate of the core is $1x10^{-5}$ M_{Earth} yr^{-1}, then the planet takes ~$4x10^{6}$ years to form and has a final high-Z mass of ~40 M_{Earth}.

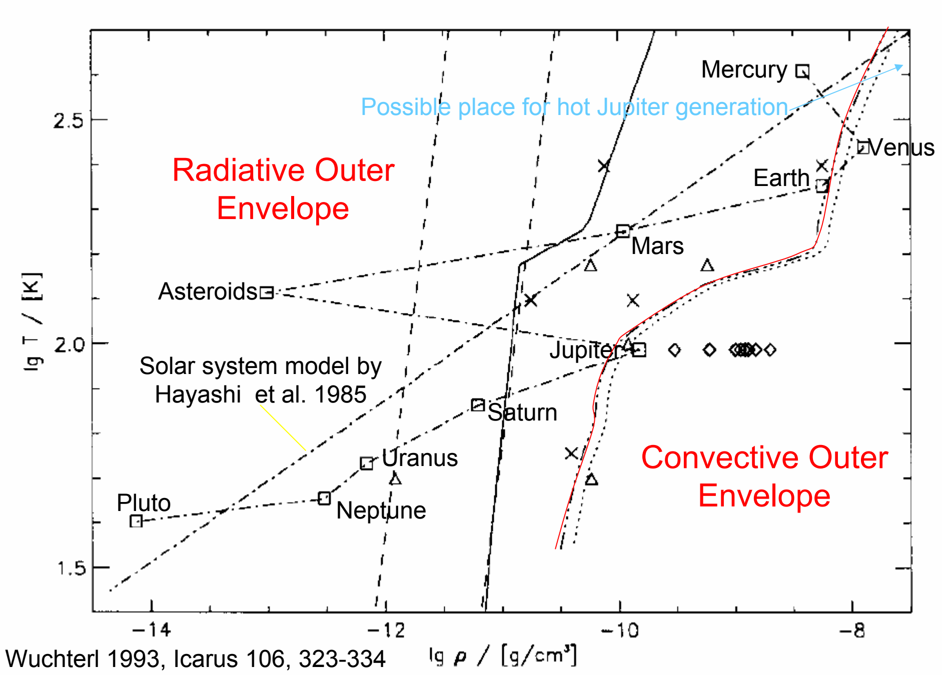
Hydrodynamic models of Giant Planet Formation Near Stars

Radiative outer envelopes may oscillate and therefore may prevent massive accretion.

But most extrasolar planets have masses > 0.5 MJ_{up}. They probably require efficient gas accretion and therefore should satisfy the convective outer envelope criterion.

The situation is illustrated in the next slide which shows the stability border (caused by specific luminosity L/M) in a temperature density diagram. Several model disks, one with positions of planets indicated, are overplotted. With improved numerical treatment of opacities and convection the stability border is likely to move more to the left.

Convective radiation hydrodynamical calculations of core-envelope growth at 0.05 AU, for particle-in-a-box core mass accretion at nebula temperatures of 1250 and 600 K, show gas accretion beyond 300 M_{Earth} at core masses of 13.5 M_{Earth} and 7.5 M_{Earth}, respectively (Wuchterl 1996, 1997).



Conclusions of the chapter by Wuchterl on Giant Planet Formation

Jupiter and Saturn are composed primarily of hydrogen and helium, yet the heavy elements may hold the key to their formation.

They have more heavy elements than were present in the protosolar gas. Were the heavy elements the first to accrete, or did the enrichment occur at later stages? Depending on this Jupiter and Saturn may have received very different amounts of planetesimals and may have formed either very rapidly (such as through the nebula instability mechanism) or more slowly (such as through nucleated instability).

Three possibilities exist for the difference between Jupiter and Saturn on one hand and Uranus and Neptune on the other hand. In Uranus and Neptune ...

1. gas accretion was limited to ~1 M_{Earth} by a hydrodynamic instability caused mainly by low gas density.
2. The cores of these planets grew more slowly because they did not achieve sufficient mass to accrete large quantities of gas before the solar nebula was dispersed.
3. The gas in the Uranus/Neptune region was dispersed rapidly via photoevaporation, before Uranus and Neptune were formed.

The nucleated instability hypothesis may explain the formation of giant planets in our and other solar systems. Presently known extrasolar planets close to their parent star may have accreted in situ.

Eiichiro Kokubo, Planetary accretion: From Planetesimals to Protoplanets, Rev. Mod. Astronomy 14, 117-132, 2001.

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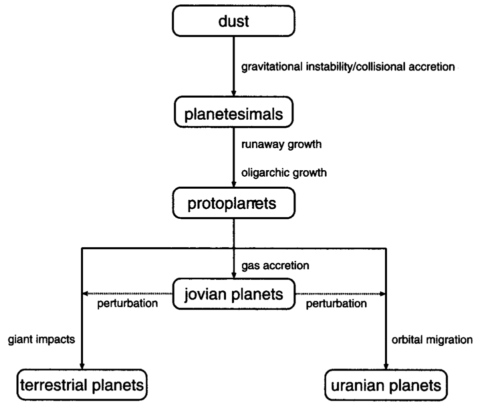

Figure 7: Flow chart of planet formation.