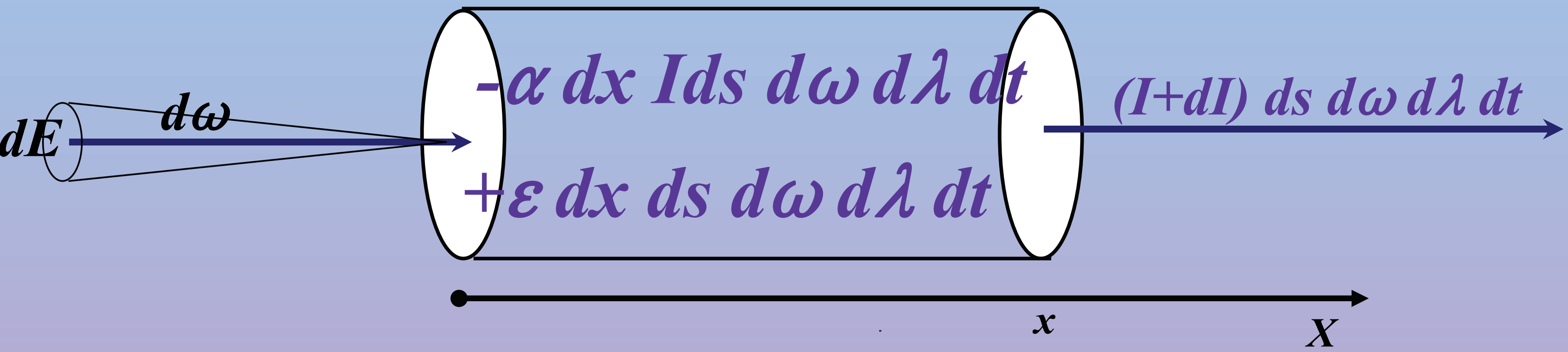
Basics of radiative transfer

Radiative transfer equation

$dE = I \, ds \, d\omega \, d\lambda \, dt$ – definition of intensity



$$dI / dx = -\alpha I + \varepsilon$$

or

$$dI / d\tau = -I + S$$

α - volume extinction coefficient

ε - volume emission coefficient

$d\tau = \alpha dx$ – optical depth

$S = \varepsilon / \alpha$ - source function

Basics of radiative transfer

1. **General solution**

   $$I(x) = I_0 e^{-\int_0^x \alpha(x')dx'} + \int_0^x \varepsilon(x') e^{-\int_{x'}^x \alpha(t)dt} dx'$$

2. **Empty space ($\alpha=0$, $\varepsilon=0$)**

   $I(x)=const$

3. **Medium without sources ($\alpha \neq 0$, $\varepsilon=0$) (Beer's law)**

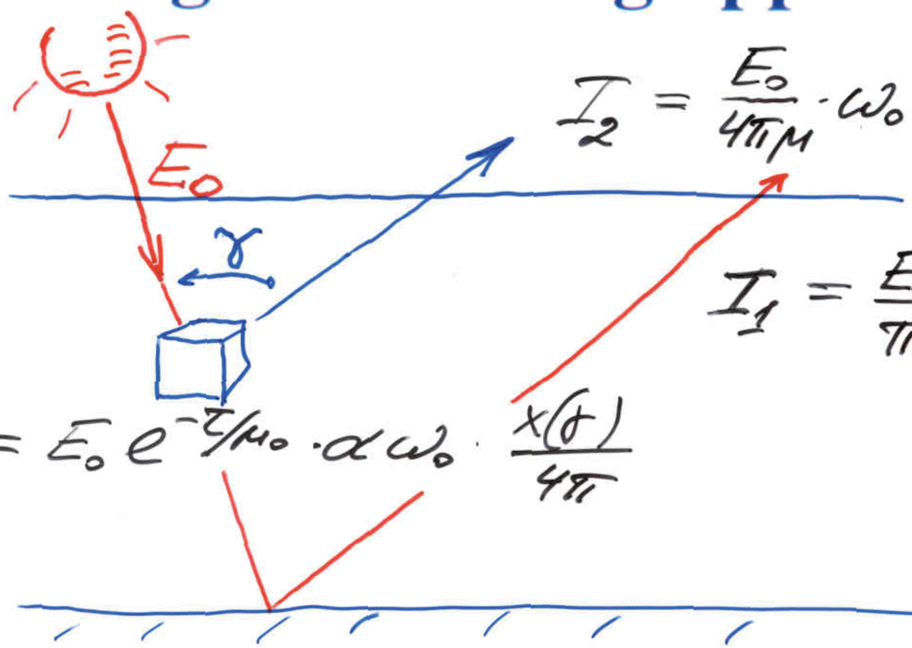
   $$I(x) = I_0 e^{-\int_0^x \alpha(t)dt}$$

Scattered solar light in the atmosphere

1. Radiative transfer equation

$$\mu \frac{dI}{dx} = -\alpha I + \varepsilon$$

2. Single scattering approximation (τ << 1)

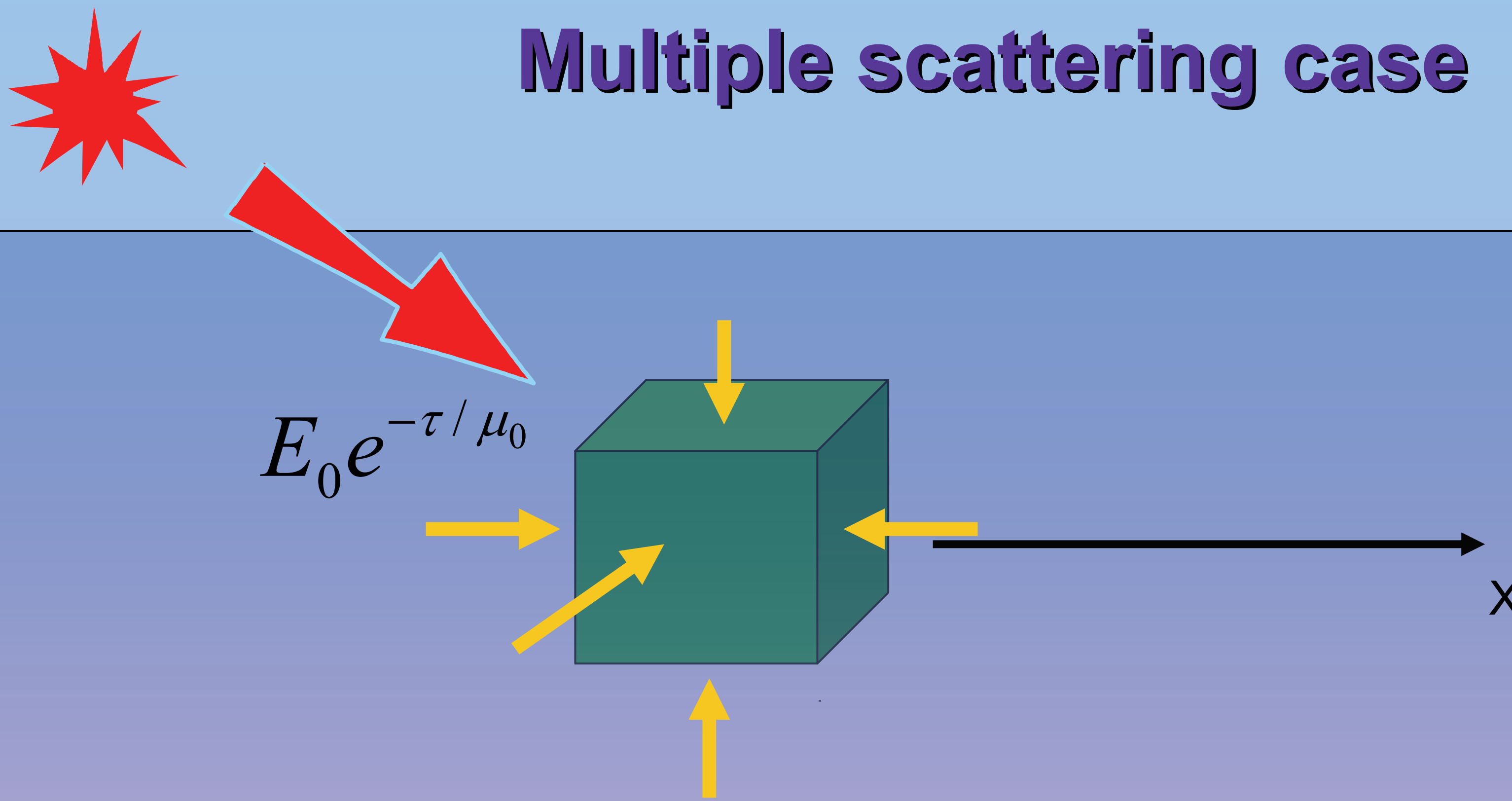


$$I_2 = \frac{E_0}{4\pi\mu} \cdot \omega_0 x(\gamma) \int_0^\infty \alpha(z) \cdot e^{-\tau(z)\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right)} dz$$

$$I_1 = \frac{E_0}{\pi} A_S \mu_0 e^{-\tau_0\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right)}$$

$$\varepsilon = E_0 e^{-\tau/\mu_0} \cdot \alpha \omega_0 \cdot \frac{x(\gamma)}{4\pi}$$

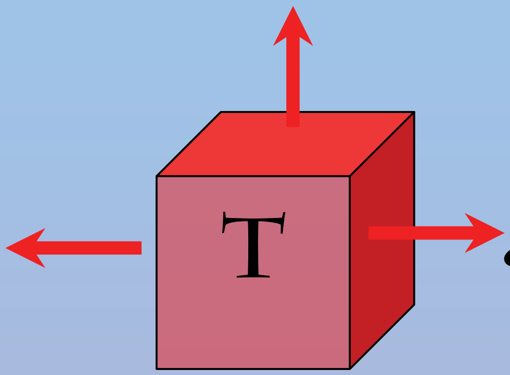
Multiple scattering case



$$\varepsilon = E_0 e^{-\tau / \mu_0} \omega_0 \alpha \frac{p(\gamma)}{4\pi} + \omega_0 \alpha \int_{\Omega} I(\omega') p(\gamma') \frac{d\omega'}{4\pi}$$

$$\frac{dI}{dx} = -\alpha I + \varepsilon$$

Thermal radiation

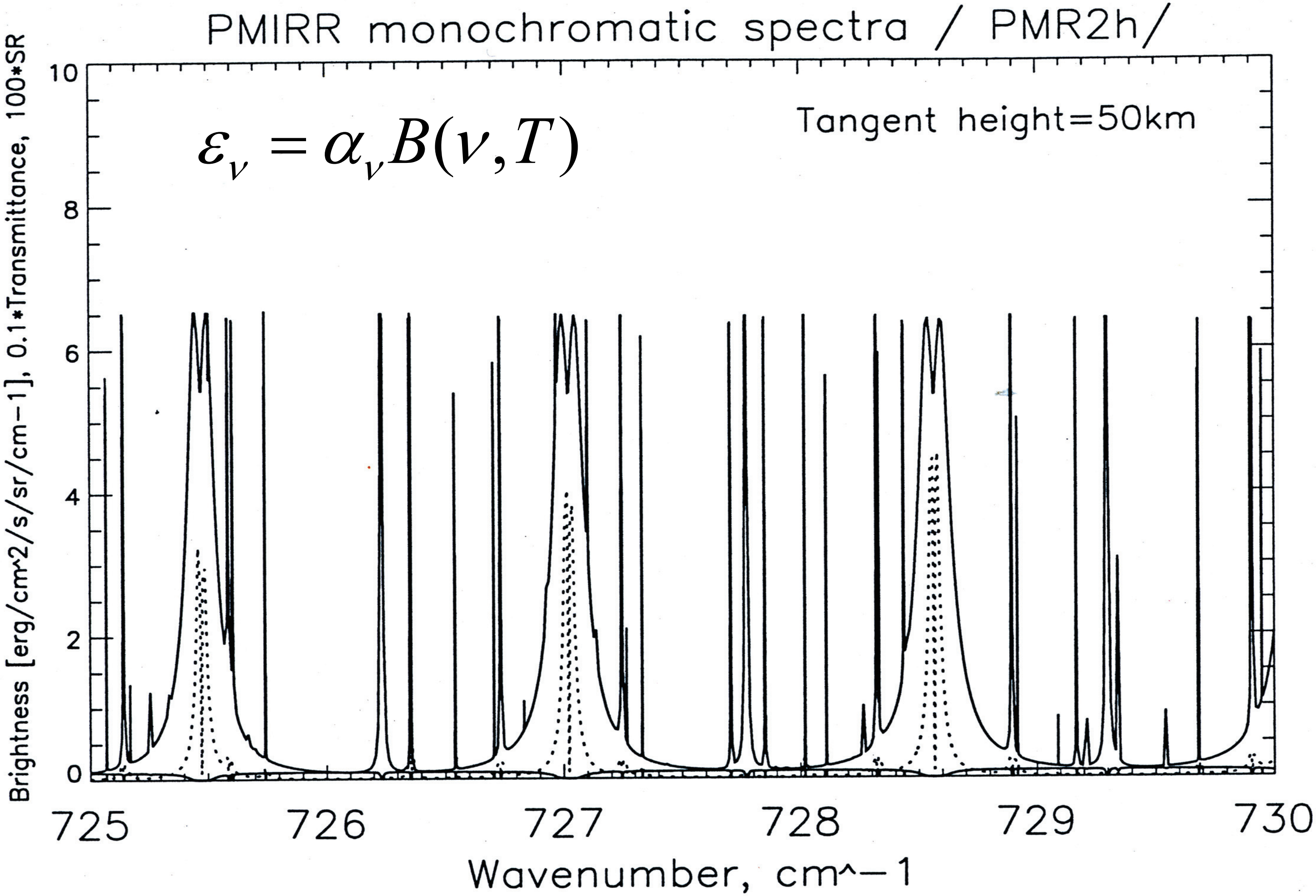


$$\varepsilon_\nu = \alpha_\nu B(\nu, T) = \alpha_\nu \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

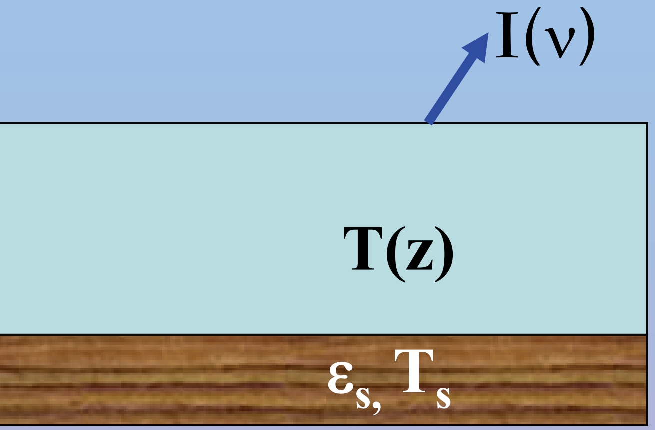
$$\Phi = \sigma T^4$$ **- full thermal flux**

- **Local thermodynamic equilibrium (LTE)**
- **LTE breakdown above relaxation level (n~10^{15} cm^{-3})**
- **Thermal radiation in gaseous atmosphere**

Atmospheric spectrum in CO_2 band



Radiative transfer equation for thermal radiation



$$I(\nu) = \varepsilon_s B_\nu(T_s)t + \int_{Surface}^{Space} B_\nu[T(z')]dt_\nu$$

Scattering is neglected

$$t_\nu(z') = \int_{z'}^{space} e^{-\tau_\nu(z)}dz$$ ***- transmittance***

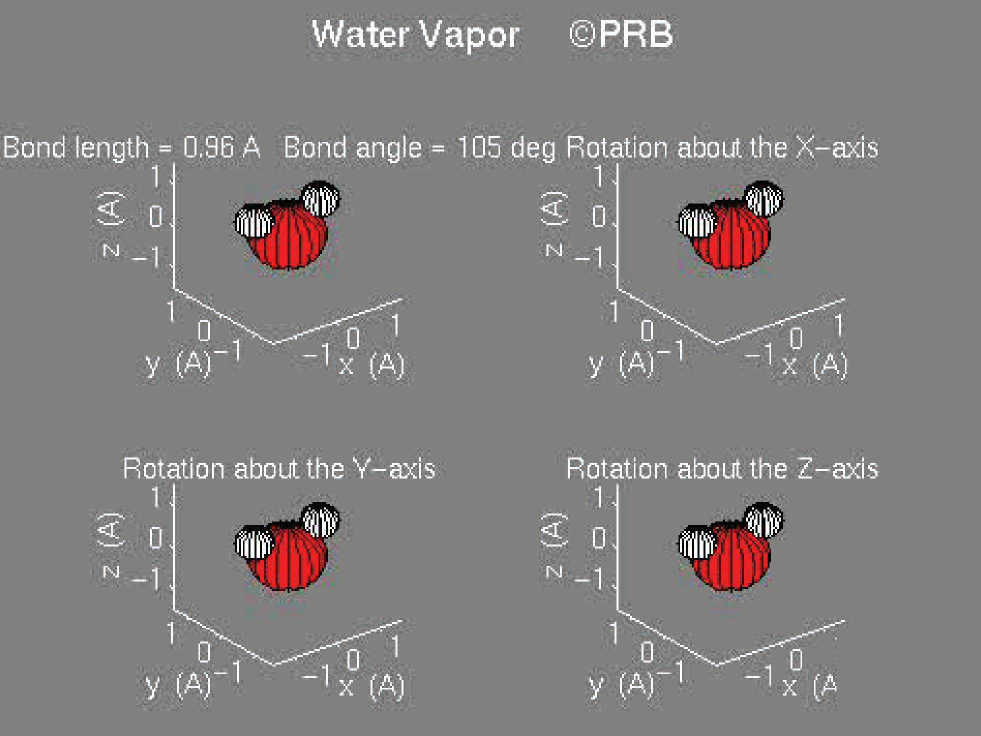
$$I(\nu) = \varepsilon_s B_\nu(T_s)t_\nu + \int_{Surface}^{Space} B_\nu[T(\xi)]\cdot K_\nu(\xi)d\xi$$

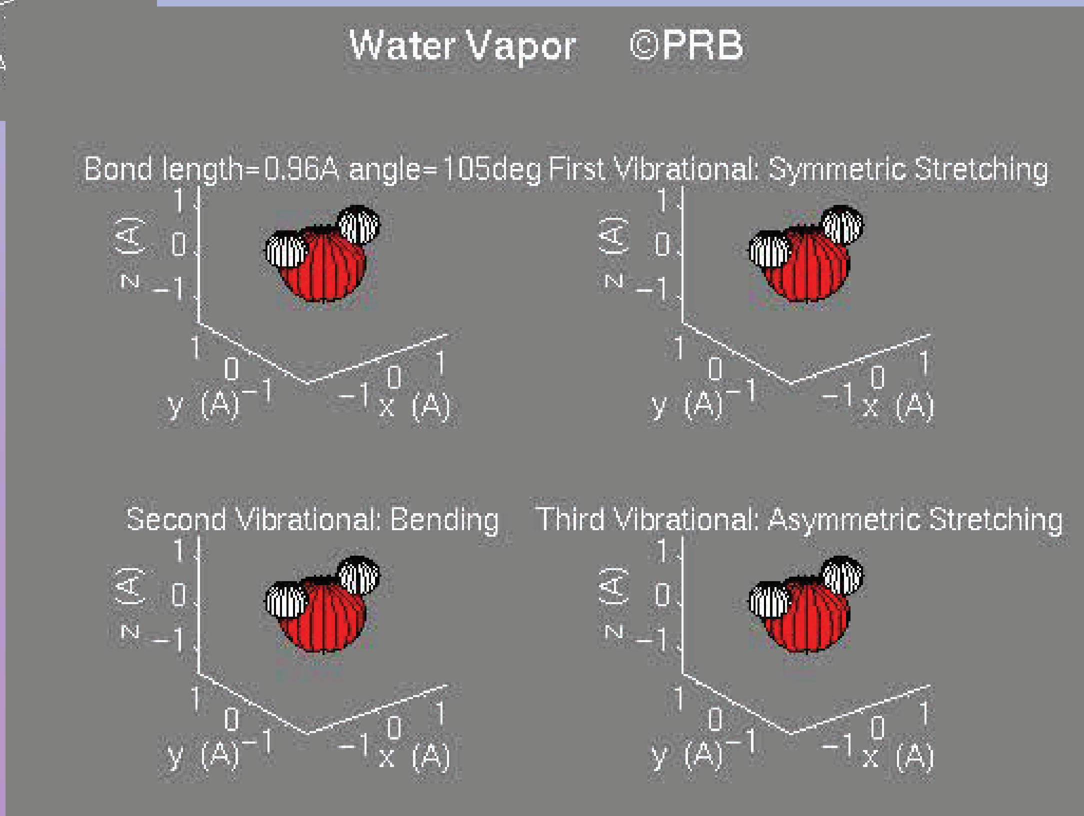
$$\xi = \lg p$$

$$K_\nu(\xi) = -\frac{\partial t_\nu}{\partial \xi}$$ - weighting function

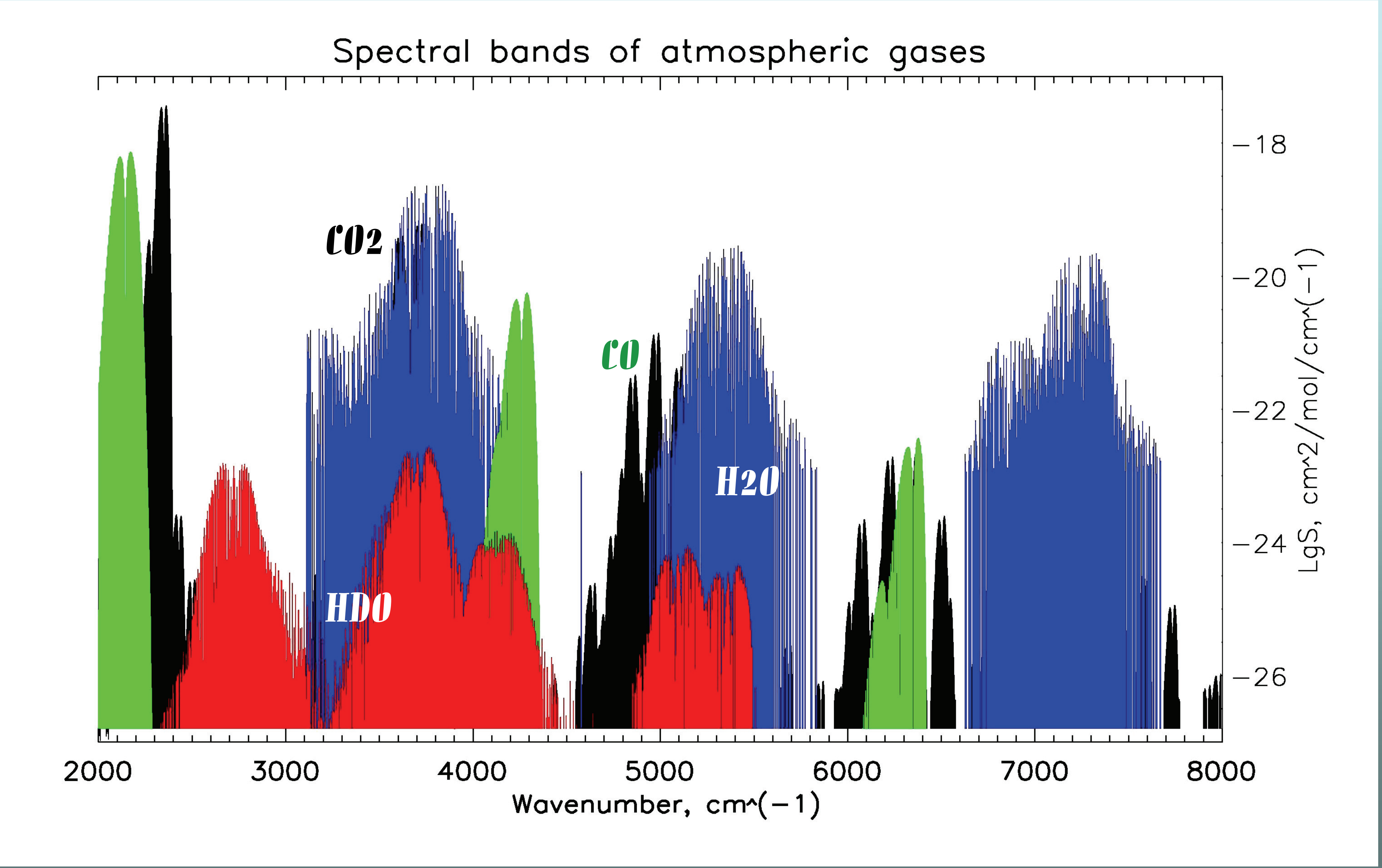
Basics of spectroscopy

Molecular "gymnastics"





Spectral bands of atmospheric gases



Spectral line shapes

1. **Spectral line shape: $k(\nu) = S \cdot L(\nu - \nu 0)$**
2. **Natural broadening: $\Delta\nu \sim 10^{-10}\ cm^{-1}$**
3. **Doppler broadening**

$$L_D = \frac{1}{\alpha_D \sqrt{\pi}} \cdot e^{-\left(\frac{\nu - \nu_0}{\alpha_D}\right)^2}$$

$$\alpha_D = \frac{\nu_0}{c}\sqrt{\frac{3kT}{m}} \simeq 10^{-2} - 10^{-4}\ cm^{-1}$$

4. **Lorentz broadening**

$$L_L = \frac{1}{\pi} \frac{\alpha_L}{(\nu - \nu_0)^2 + \alpha_L^2} \qquad \alpha_L = \alpha_0 \frac{P}{P_0}\left(\frac{T_0}{T}\right)^n$$

5. **Voigt line profile**

$$L_V = \frac{\sqrt{\ln 2}\, a}{\pi^{3/2} \alpha_D} \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{(x-y)^2 + a^2}$$

$$x = \frac{(\nu - \nu_0)\sqrt{\ln 2}}{\alpha_D} \qquad a = \frac{\alpha_L \sqrt{\ln 2}}{\alpha_D}$$

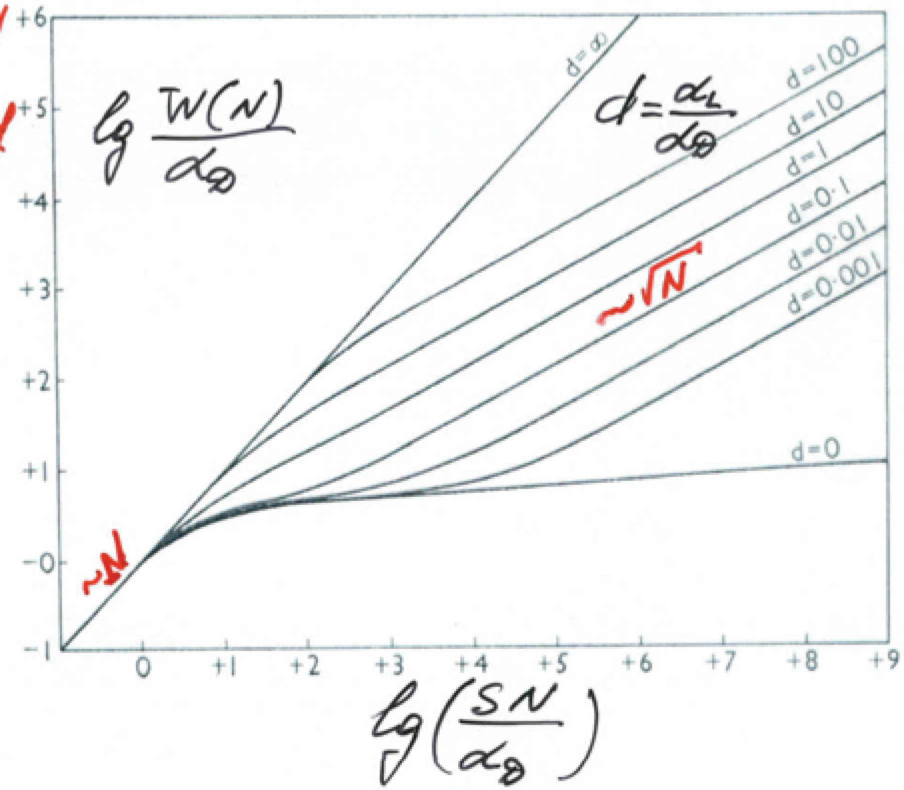
Solar range: Equivalent width and curve of grouth

1. Equivalent width
2. Absorption in isolated Lorentz line

$$W(N) = 2\pi\alpha_L \cdot L(u) \begin{cases} \to \sim u, & u \ll 1 \\ \to \sim \sqrt{u}; & u \gg 1 \end{cases}$$

$$u = \frac{SN}{2\pi\alpha_L}$$

3. Absorption in isolated Voght line



4. Elsasser band model

$$\tau(x) = 2\pi u y \frac{\operatorname{sh} 2\pi y}{\operatorname{ch} 2\pi y - \cos 2\pi x}$$

$$u = \frac{SN}{2\pi\alpha_L}, \quad x = \frac{\nu}{\delta}, \quad y = \frac{\alpha_L}{\delta}$$

Solar reflected spectrum depends mainly on gas amount and to a less extent on the temperature!

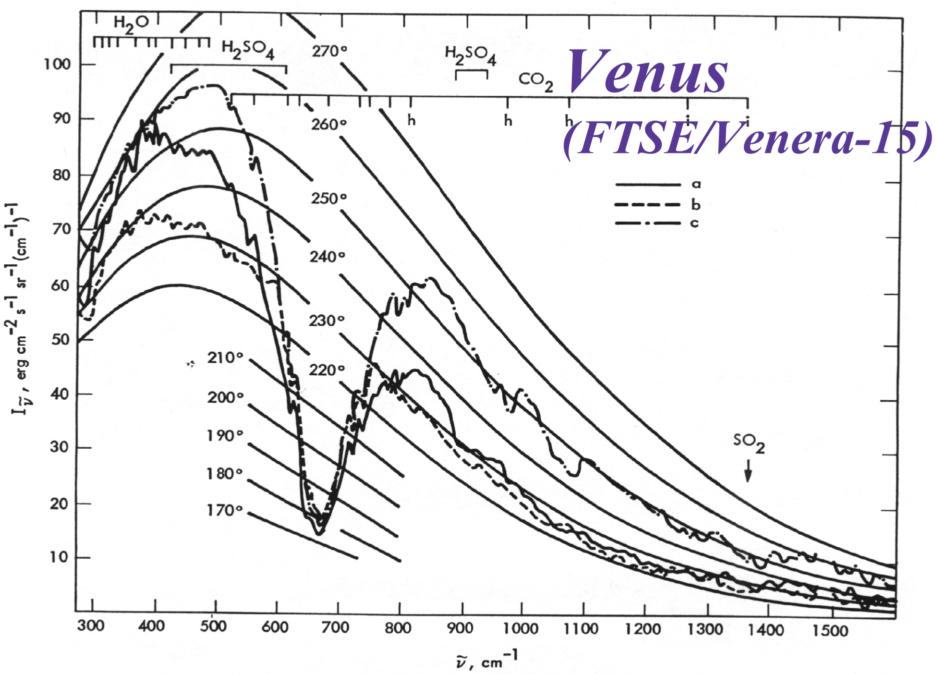
Spectra of thermal emission

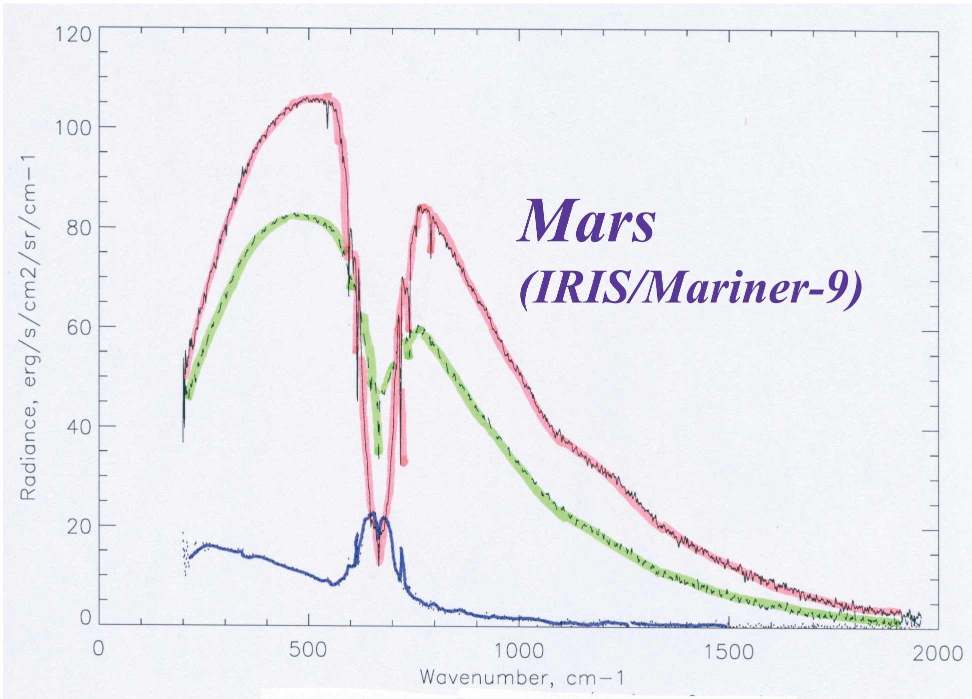
1. **Radiative transfer equation**

$$I = \varepsilon_s \cdot B(T_s) \cdot t + \int_{p_s}^{0} B[T(s')]\, dt$$

$$I = \varepsilon_s B(T_s)\, t + \int_{-\infty}^{\infty} B[T(\xi)]\, K(\xi)\, d\xi \qquad \xi = \lg P$$

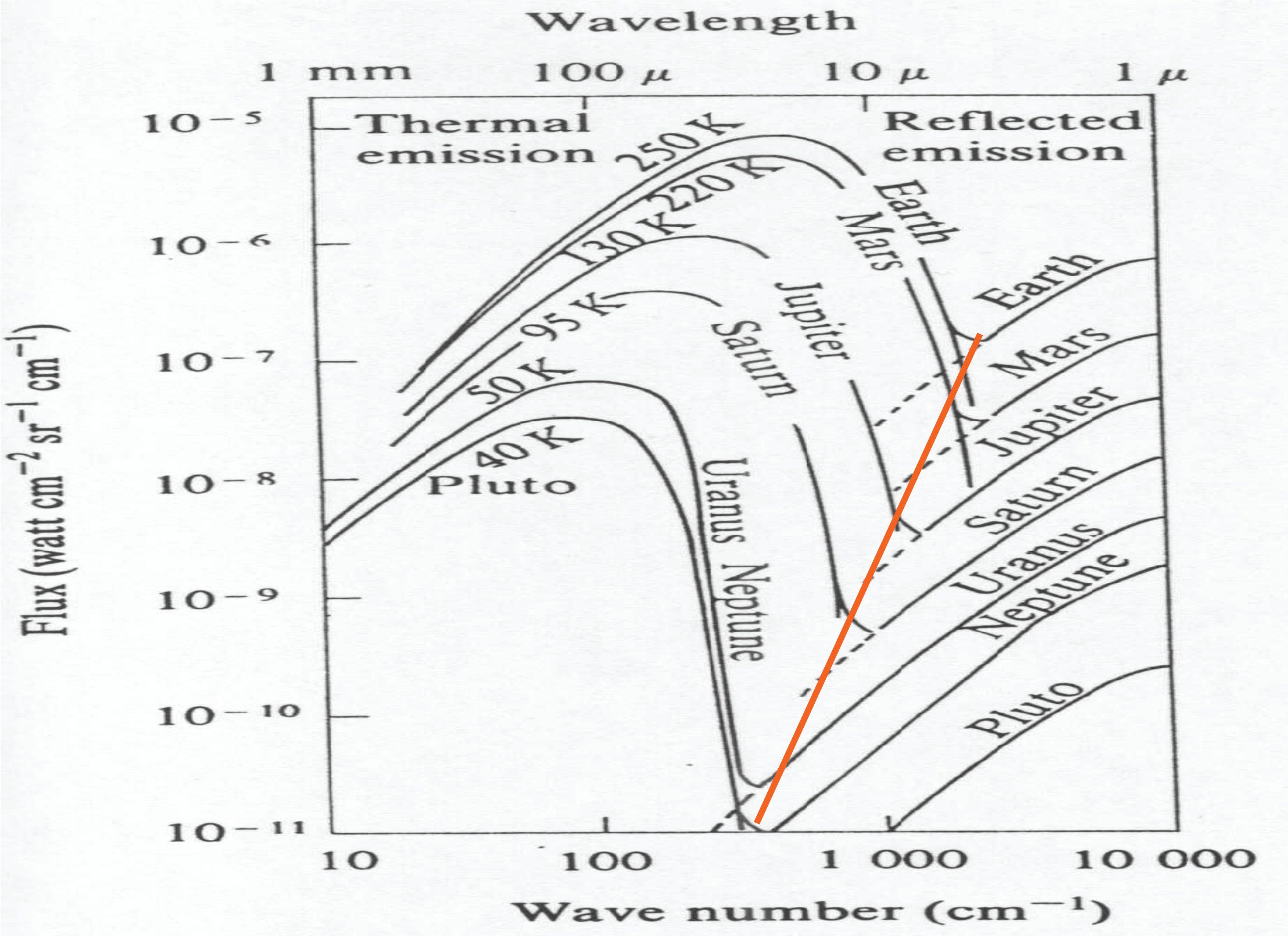
2. **Weighting function** $K(\xi) = -\frac{\partial t}{\partial \xi}$
3. **Examples of thermal spectra of planets**





Thermal spectrum depends temperature and gas abundance!

General shape of the planetary spectra

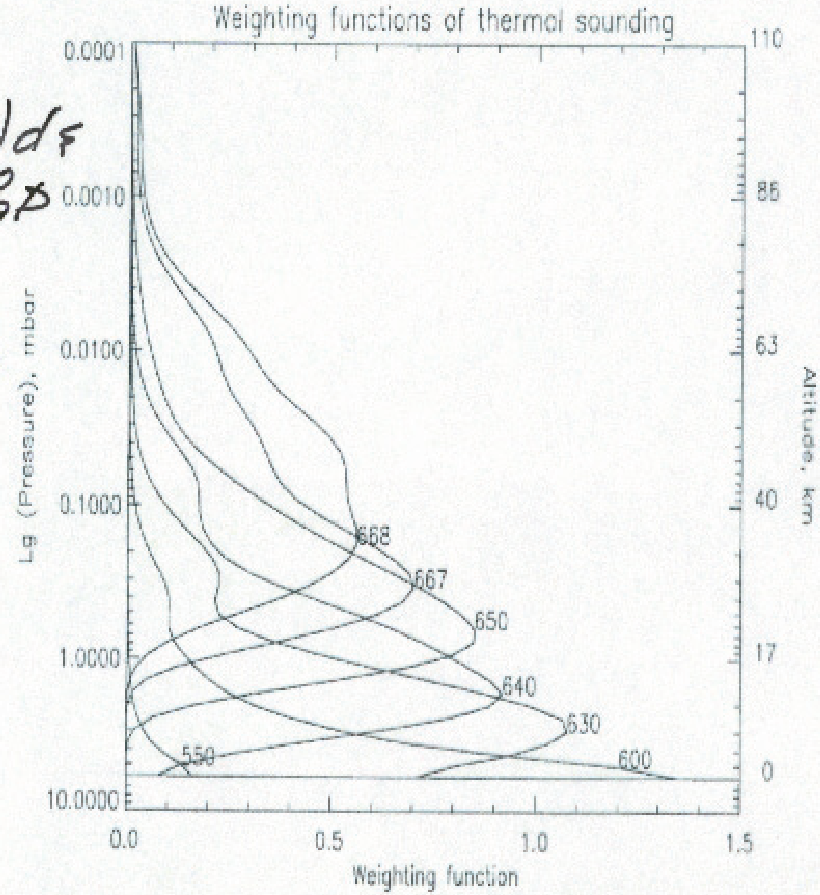


Additional slides to Basics of Radiative Transfer

Principles of temperature sounding

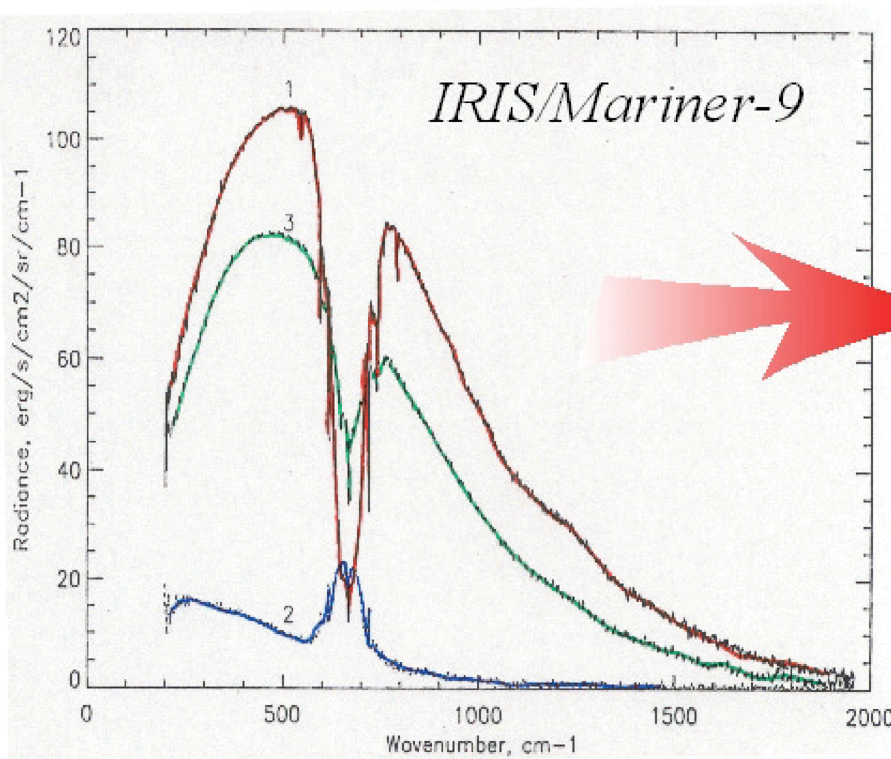
- In strong bands of atmospheric gases thermal radiation forms at different altitudes depending on wavelength
- $$I(\nu) = \varepsilon_S B(T_S) t_{ATM} + \int_{-\infty}^{\infty} B[T(\xi)] \cdot K(\xi) d\xi$$
  $$K(\xi) = -\frac{\partial t}{\partial \xi} \qquad \xi = \lg p$$
- Gas should be well mixed and its abundance known
- Local thermodynamic equilibrium
- Temperature retrieval is an ill-posed problem

Weighting functions of thermal sounding at Mars

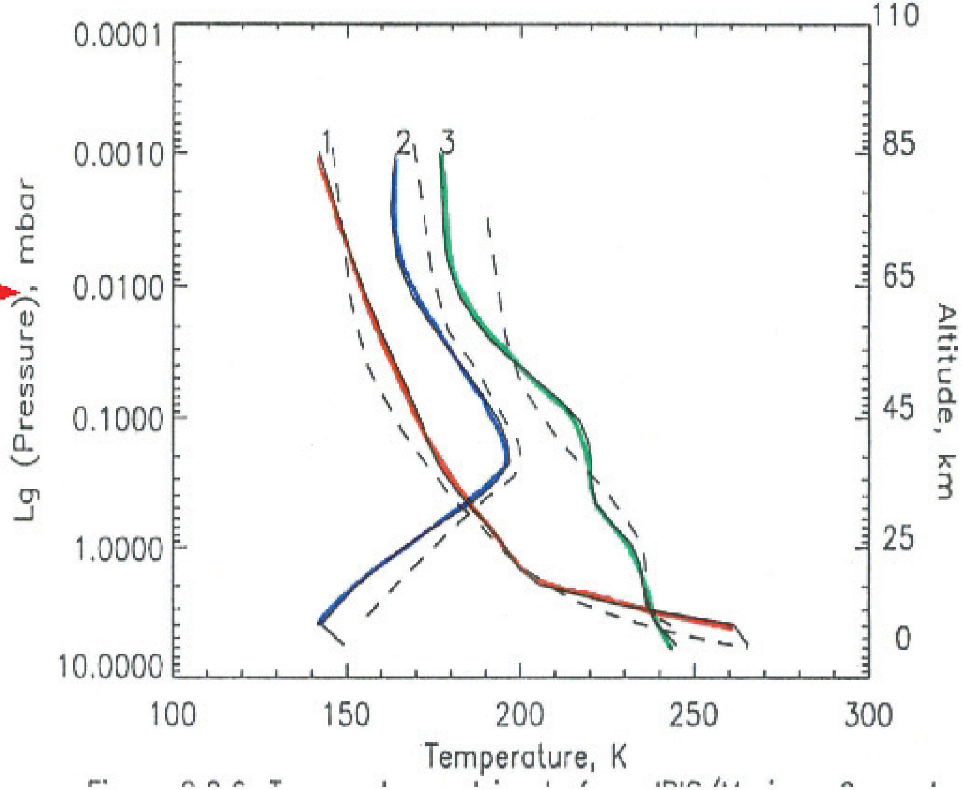


Example of temperature sounding

Measured spectra of Mars



Retrieved temperature profiles



Scattered solar light in the atmosphere (2)

5. Two stream approximation /Schwarzschild- Schuster/

$$I^{\uparrow}(\tau) = \int I(\tau) \sin\theta \, d\theta \qquad I^{\downarrow}(\tau) = \int I(\tau) \sin\theta \, d\theta$$

$$\begin{cases} \frac{1}{2} \frac{dI^{\uparrow}}{d\tau} = I^{\uparrow} - S \\ -\frac{1}{2} \frac{dI^{\downarrow}}{d\tau} = I^{\downarrow} - S \\ S(\tau) = \frac{1}{2}\left[I^{\uparrow} + I^{\downarrow} \right] \end{cases} \Rightarrow S(\tau) = F\left(\tau + \frac{1}{2}\right)$$

6. N-stream approximation

$$\mu_i \frac{dI(\tau,\mu_i)}{d\tau} = I(\tau,\mu_i) - \frac{1}{2} \sum a_j I(\tau,\mu_i) \qquad i = \pm 1 \ldots \pm N$$

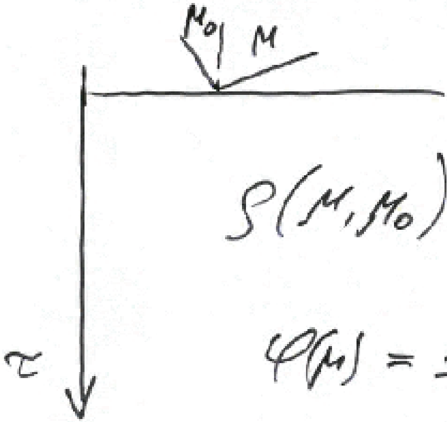
Scattered solar light in the atmosphere (3)

3. Spherical phase function X(γ)=1 /Hvolson/

$$\begin{cases} \mu \dfrac{dI}{d\tau} = I - S \\ S = \dfrac{\omega_0}{2}\displaystyle\int_{-1}^{1} I(\mu)\,d\mu + \dfrac{\omega_0}{4\pi} E_0 e^{-\tau/\mu_0} \end{cases}$$

$$\Rightarrow S(\tau) = \frac{\omega_0}{2}\int_0^{\infty} Ei_1(|\tau - t|)\, S(t)\, dt + \frac{\omega_0}{4\pi} E_0 e^{-\tau/\mu_0}; \quad Ei_1(x) = \int_1^{\infty} \frac{e^{-xy}}{y}\, dy$$

4. Reflection from semi-infinite atmosphere /Ambartsumjan/

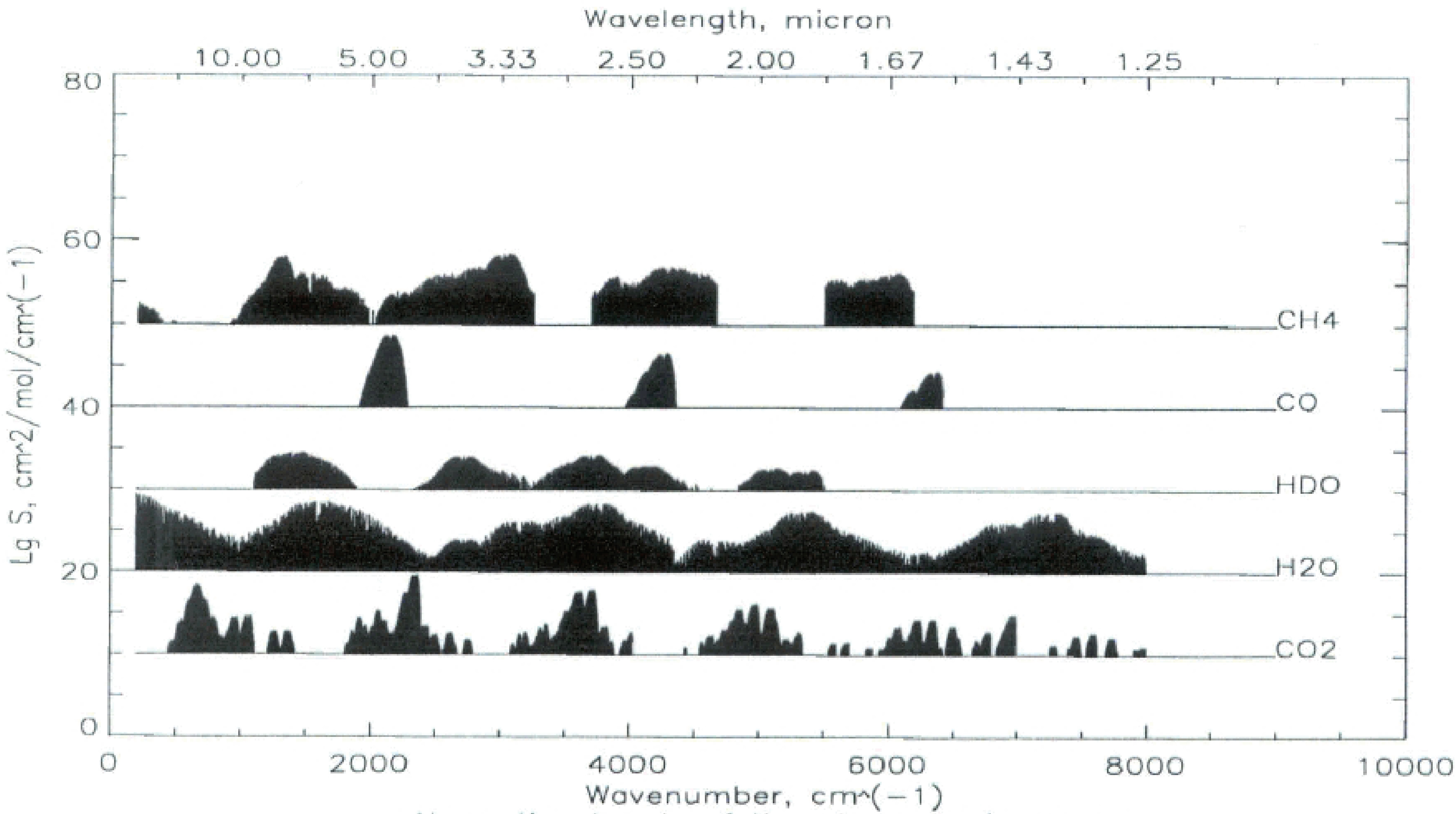


$$\rho(\mu, \mu_0) = \frac{\omega_0}{4} \frac{\varphi(\mu)\cdot\varphi(\mu_0)}{\mu + \mu_0}$$

$$\varphi(\mu) = 1 + \frac{\omega_0}{2}\varphi(\mu)\int_0^1 \frac{\varphi(\mu')}{\mu + \mu'}\, d\mu'$$

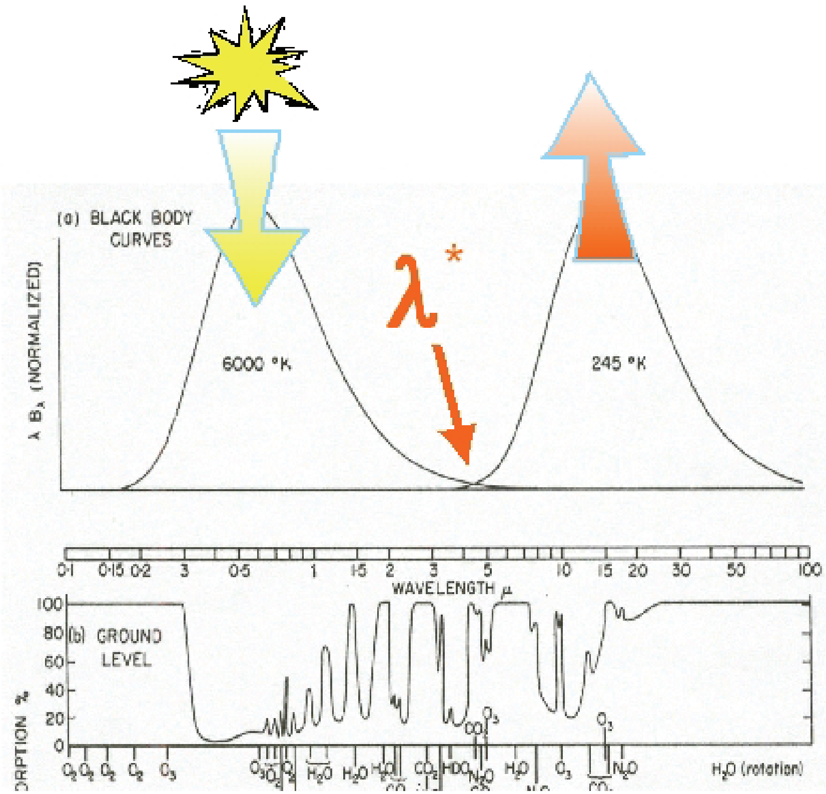
Basics of spectroscopy (3)

Absorption bands of atmospheric gases

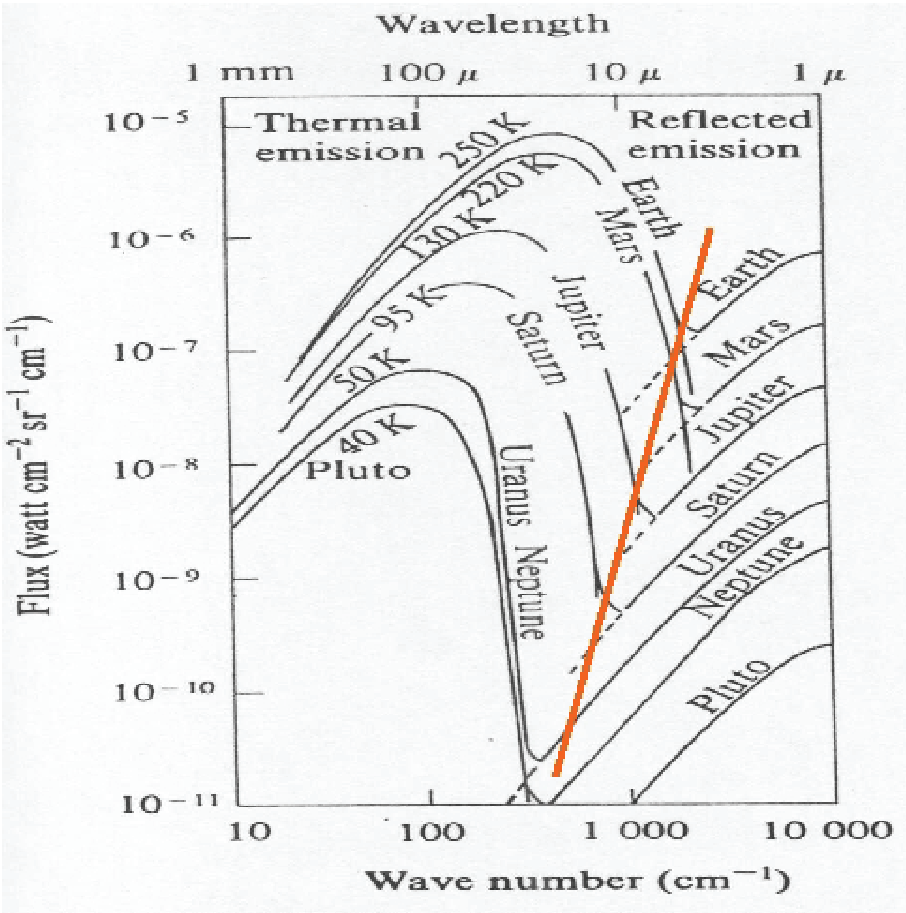


Absorption bands of the atmospheric gases

Energy balance of planets



Planetary spectra



Effective temperature

$$E(1-A) = 4\sigma T^4_{eff}$$

Solar constant

$E_\oplus = 1370\ W/m^2$