2. Gravity and rotation

2.1 Description of gravity field

Gravitational potential V

Gravity (acceleration) $\vec{g} = -\operatorname{grad} V$

Point mass : $V = -\frac{GM}{r}$. (also spherically symmetric body)

Ellipsoid : $V = -\frac{GM}{r} \cdot \left(1 + \left(\frac{a}{r}\right)^2 \cdot J_2 \cdot P_2(\cos\vartheta) \ldots \right)$

a : equatorial radius $\qquad P_2(\cos\vartheta) = \frac{3}{2}\cos^2\vartheta - \frac{1}{2}$

General : $V = -\frac{GM}{r}\left(1 + \sum_{l=2}^{\infty} \left(\frac{a}{r}\right)^l \sum_{m=0}^{l} P_l^m(\cos\vartheta) \times \ldots\right.$

$\left.\ldots \left(C_l^m \cos m\varphi + S_l^m \sin m\varphi\right)\right)$

ϑ : Colatitude $\qquad \varphi$: Longitude

2.2 Determination of gravity field

- Surface measurements of g (Earth)
- orbits of natural satellites, mutual perturbations of planetary orbits
- orbits of space probes, radio doppler tracking
- radar altimetry of ocean surface (Earth)

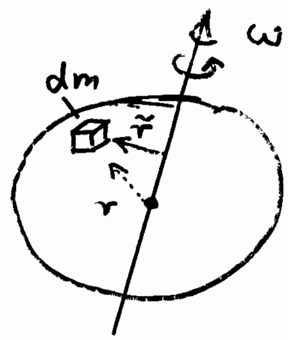
2.3 Moment of inertia and inferences on internal structure

Total mass M $\rightarrow$ mean density $\bar{\rho} = \dfrac{M}{\frac{4\pi}{3} a^3}$

(no clue on internal density stratification)

Mass $M = \int dm$

Moment of inertia $I = \int \tilde{r}^2 dm$



For non-spherical bodies is I a tensor

$\vec{L} = \underline{\underline{I}} \cdot \vec{\omega}$ L : angular momentum
ω : frequency of rotation

3 principal components (preferred axes)

C (largest), B, A (smallest)

Formula of McCullagh :

general $J_2 = \dfrac{C - \frac{1}{2}(A+B)}{Ma^2}$

ellipsoid $J_2 = \dfrac{C-A}{Ma^2}$

wanted : C or C/Ma^2

Dynamical ellipticity $H = \dfrac{C-A}{C}$

Can be uniquely determined from

- Precession data
- Nutation data (body with locked rotation)

$\Rightarrow \dfrac{C}{Ma^2} = J_2/H$

In case of hydrostatic equilibrium:

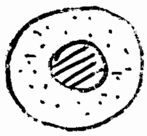
Flattening $f \simeq \frac{3}{2} J_2 + \frac{1}{2} m$

$$m = \frac{\omega^2 a^3}{GM} \qquad \left(\frac{\text{Equatorial centrifugal force}}{\text{Gravity}} \right)$$

$$\frac{C}{Ma^2} \simeq \frac{2}{3} \left[1 - \frac{2}{5} \left(\frac{4m - 3J_2}{m + 3J_2} \right)^{1/2} \right]$$

(Radau - Darwin formula)

Moment of inertia factor : Examples

		C/Ma^2
	homogeneous sphere	0.4
	thin hollow spherical shell	0.666..
	core radius $a_k = \frac{a}{2}$, density $\rho_k = 2\rho_M$:	0.347
	$\rho_k = 10\rho_M$	0.241

2.4 Models for terrestrial planets / Jovian satellites

	Earth	Moon	Mercury	Venus	Mars	
$\bar{\rho}$	5515	3341	5430	5245	3935	$\frac{kg}{m^3}$
$\bar{\rho}_{dekompr}$	4060	3315	5280	3990	3730	
C/Ma^2	0.3308	0.390	?	?	0.366	

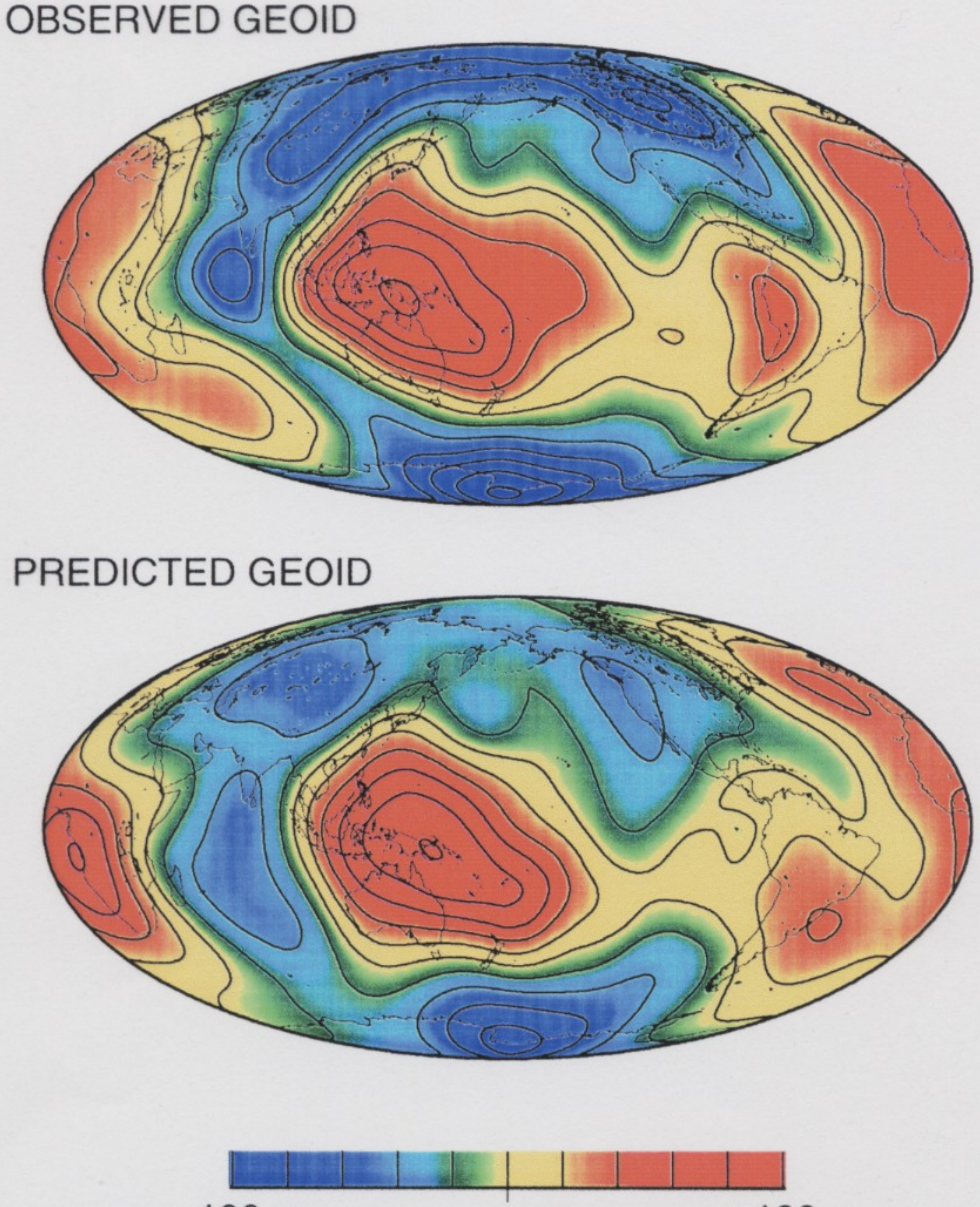
	Jo	Europa	Ganymede	Callisto	
$\bar{\rho}$	3530	3020	1940	1850	$\frac{kg}{m^3}$
C/Ma^2	0.378	0.347	0.311	0.358	

Typical zero-pressure densities of candidate materials :

Mantle rock : 3300   Fe : 7800

Crustal rock : 2900   FeS : 4900

H_2O (ice) : 920


OBSERVED GEOID
PREDICTED GEOID
-100 m
0
+100 m

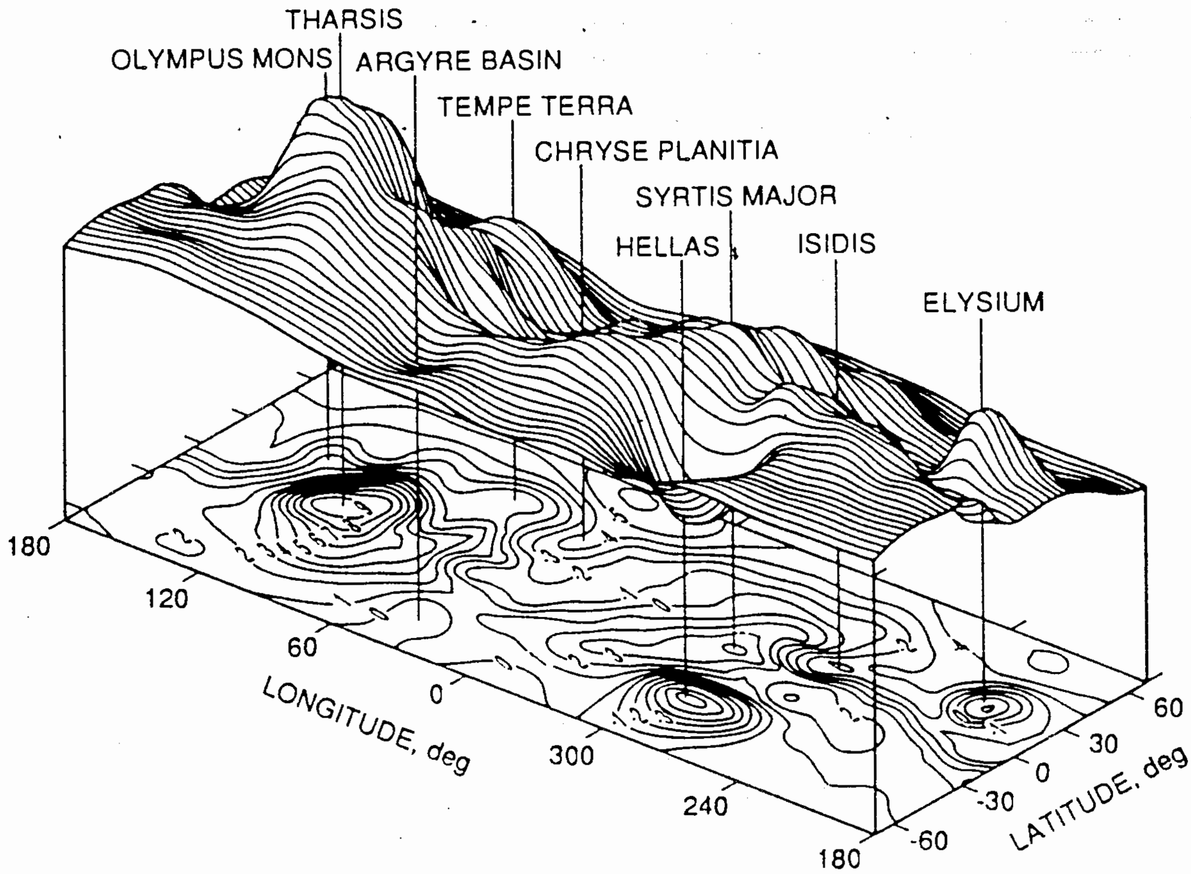


Fig. 11. Mars topography, as represented by the 16 × 16 field of Bills and Ferrari (1978) with

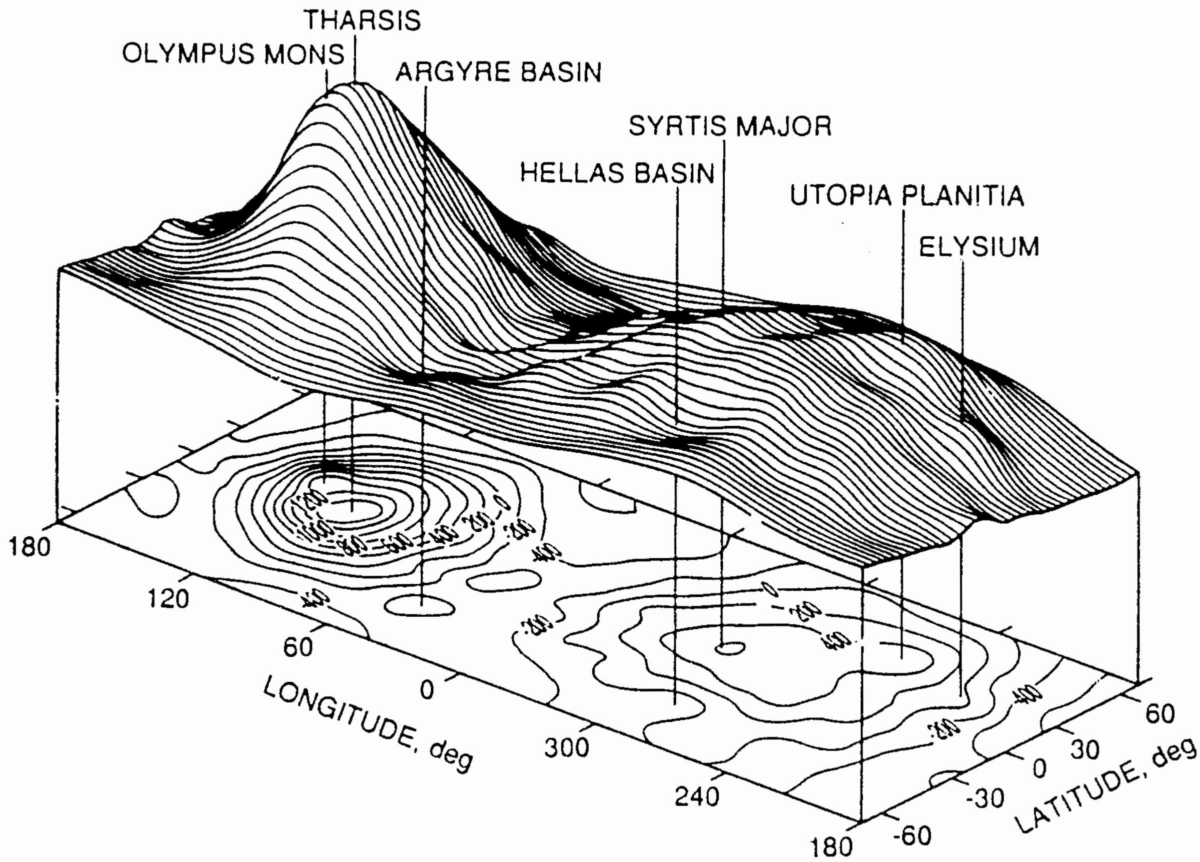


Fig. 10. Mars geoid deduced from the Balmino et al. (1982) gravity model truncated to a 16 × 16 field. A hydrostatic contribution, corresponding to $I/MR^2 = 0.366$, has been removed. The nonhydrostatic coefficients used here are the same as those given by Balmino et al. (1982) except $\bar{C}_{20} = -6.38 \times 10^{-5}$ and $\bar{C}_{40} = 0.36 \times 10^{-5}$. This Mercator projection covers 180°W to 180°E longitude, 65°S to 65°N latitude with contour intervals of 200 m.

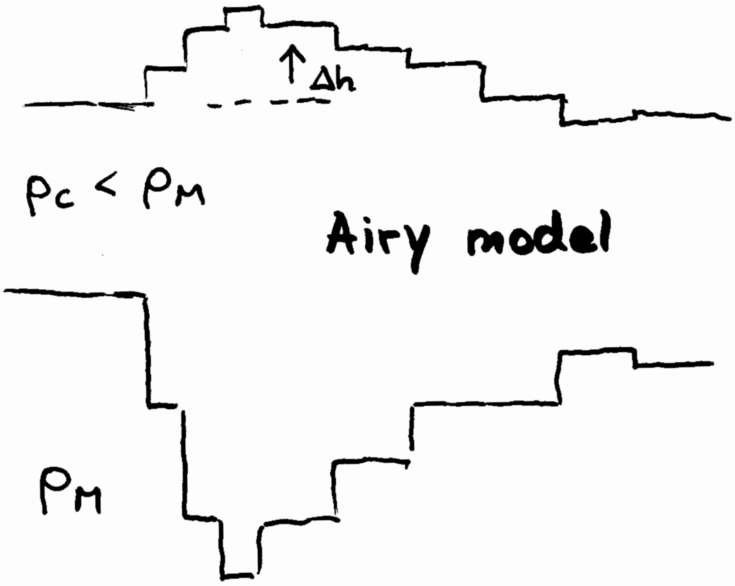
Gravity anomaly Δg :

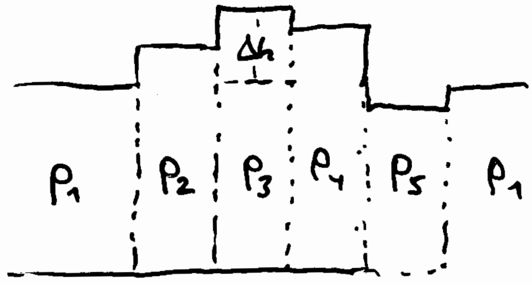
Deviation of observed gravity from theoretical gravity of a rotating ellipsoidal body (i.e. including the J_2 - term)

Geoid anomoly ΔN:

Deviation of an equipotential surface from ~~the~~ a reference ellipsoid (zero in complete hydrostatic equilibrium)

Isostasy





Pratt model

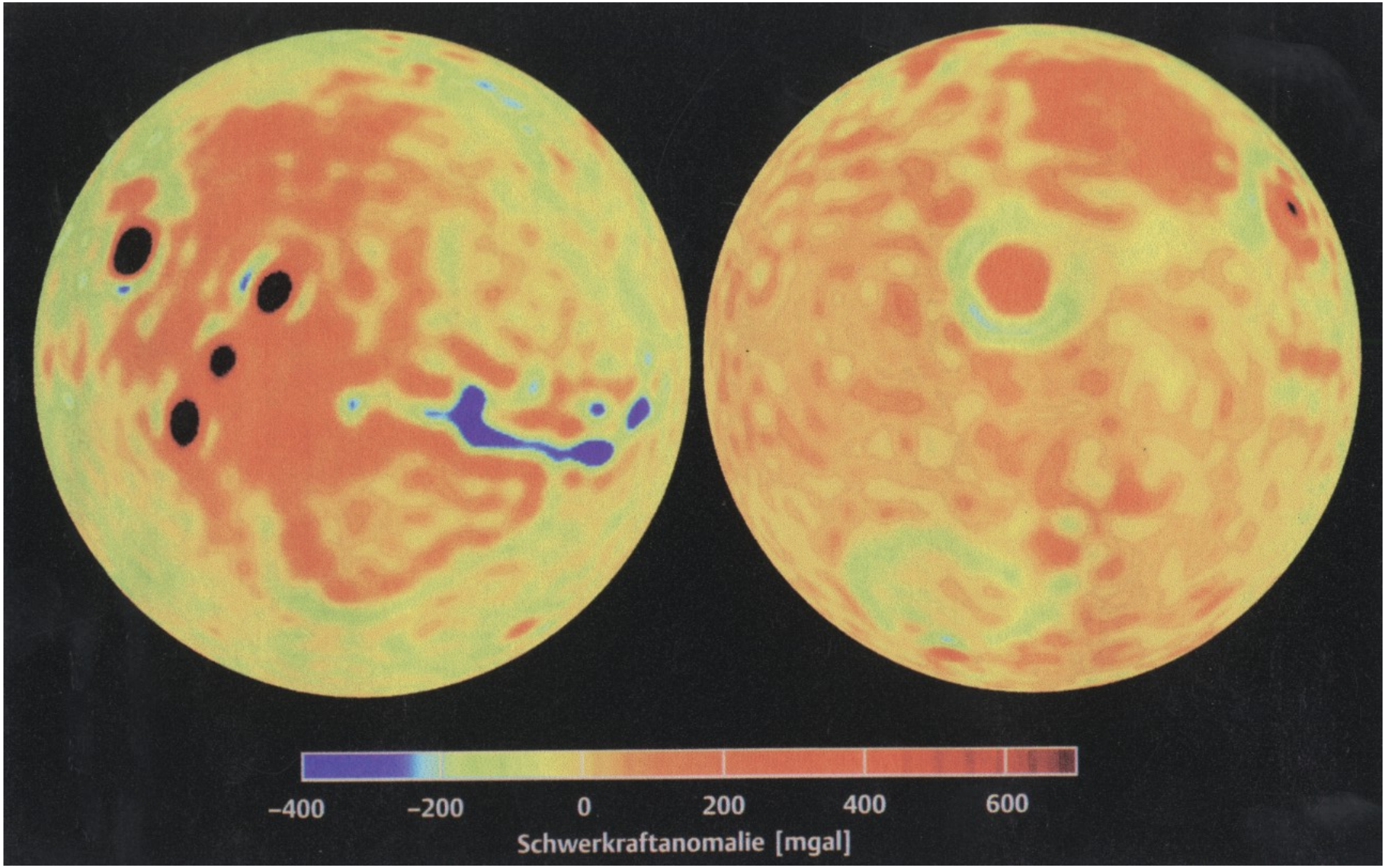
Admittance : $\dfrac{\Delta g}{\Delta h}$ or $\dfrac{\Delta N}{\Delta h}$

Rule of density moment for isostatically compensated topography

$$\Delta N = -\frac{2\pi G}{g_0} \int z \cdot \Delta\rho(z)\, dz$$

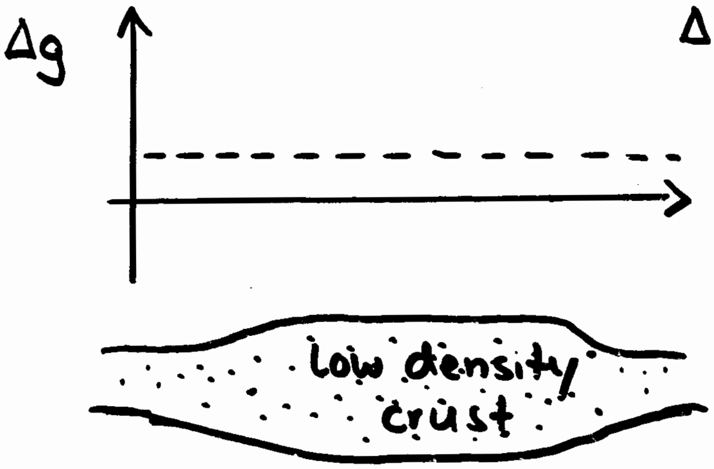
or for a well defined compensation depth z_c :

$$\frac{\Delta N}{\Delta h} = \frac{2\pi G}{g_0} \cdot \Delta\rho_{surface} \cdot z_c$$

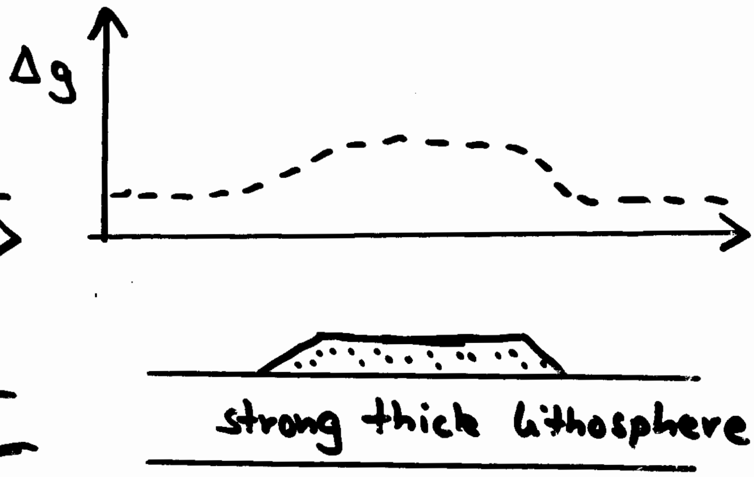


−400
−200
0
200
400
600
Schwerkraftanomalie [mgal]

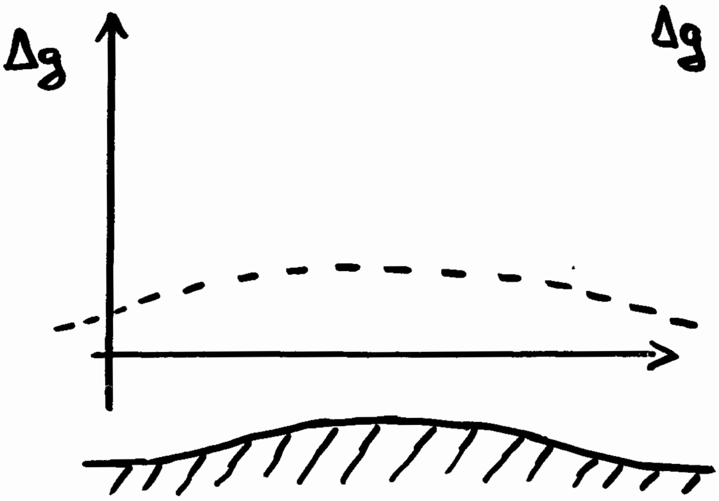
Gravity & topography at small and intermediate scales / isostasy



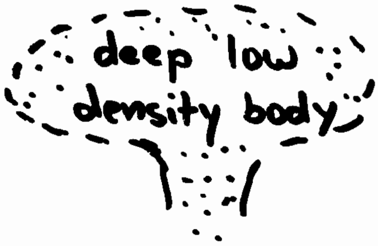
shallow compensation

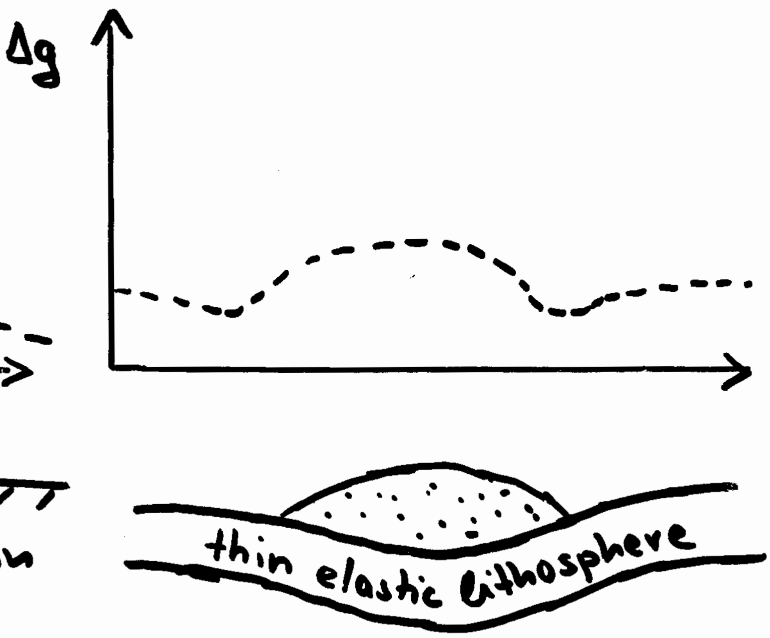


no compensation

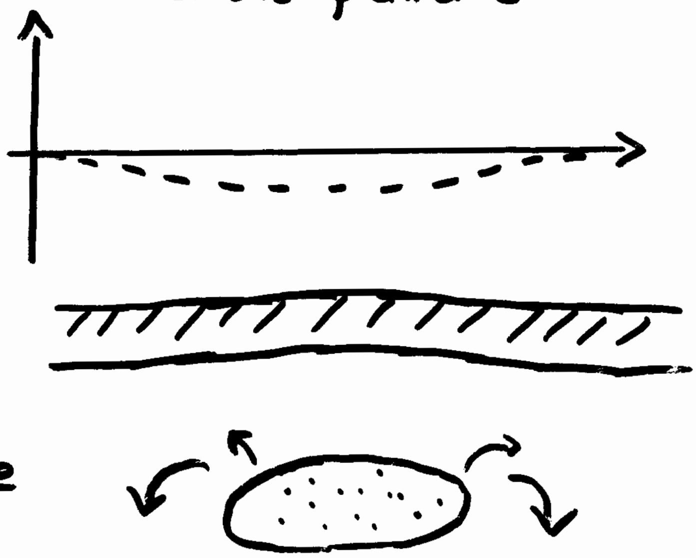


deep compensation





elastic flexure



very deep
mass anomaly
below lithosphere