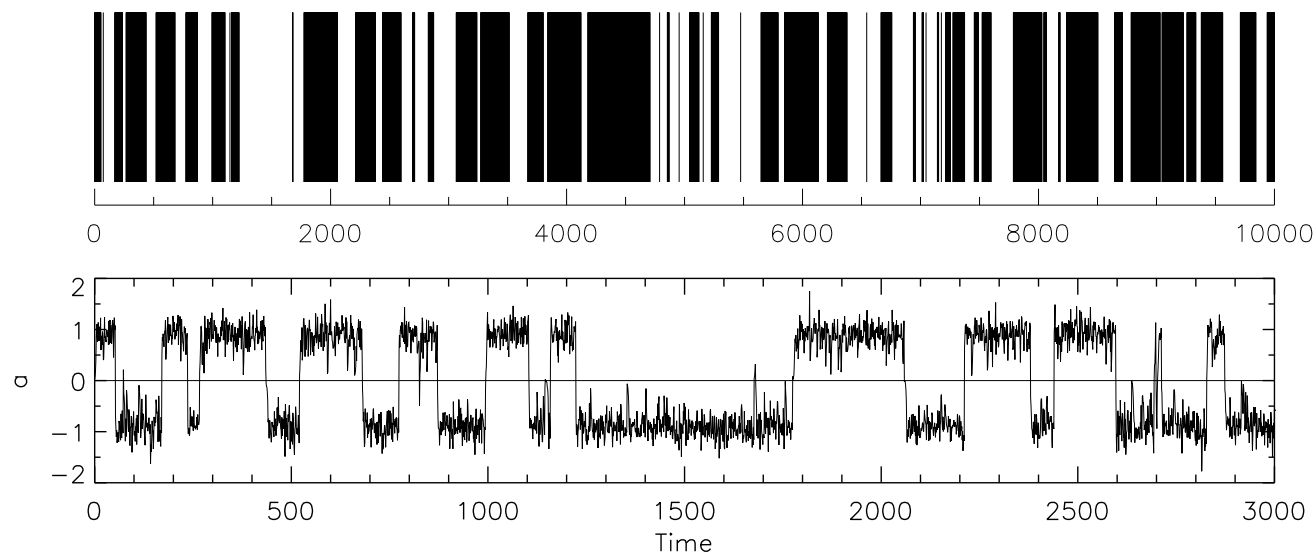


Why does the magnetic field of the Earth reverse?

Why does the
magnetic field of the
Earth reverse?

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Dieter Schmitt (Katlenburg-Lindau)



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1. Geomagnetic field

- >99% of matter in universe is plasma: gas of electrons, ions and neutral particles
- moving charged particles → electric currents
→ magnetic fields
- basic process → magnetic fields everywhere in cosmos: planets, stars, accretion disks, galaxies
- magnetic field of Earth oldest known magnetic field of a celestial body: compass, Gilbert (1600)
- Gauss (1838): spherical harmonics, sources in interior, mainly dipolar, $B_E \approx 0.3 \text{ G}$, $B_P \approx 0.6 \text{ G}$, tilt $\approx 11^\circ$, offset $\approx 450 \text{ km} \approx 0.07 R_E$, $B_D \sim r^{-3}$
- higher multipoles superposed
- secular variation
 - westdrift of nonaxisymmetric components of $0.18^\circ/\text{yr}$
 - pole wobble
 - intensity variations
- polarity reversals, $10^3 \text{ yr} \leftrightarrow 10^{5\dots7} \text{ yr}$

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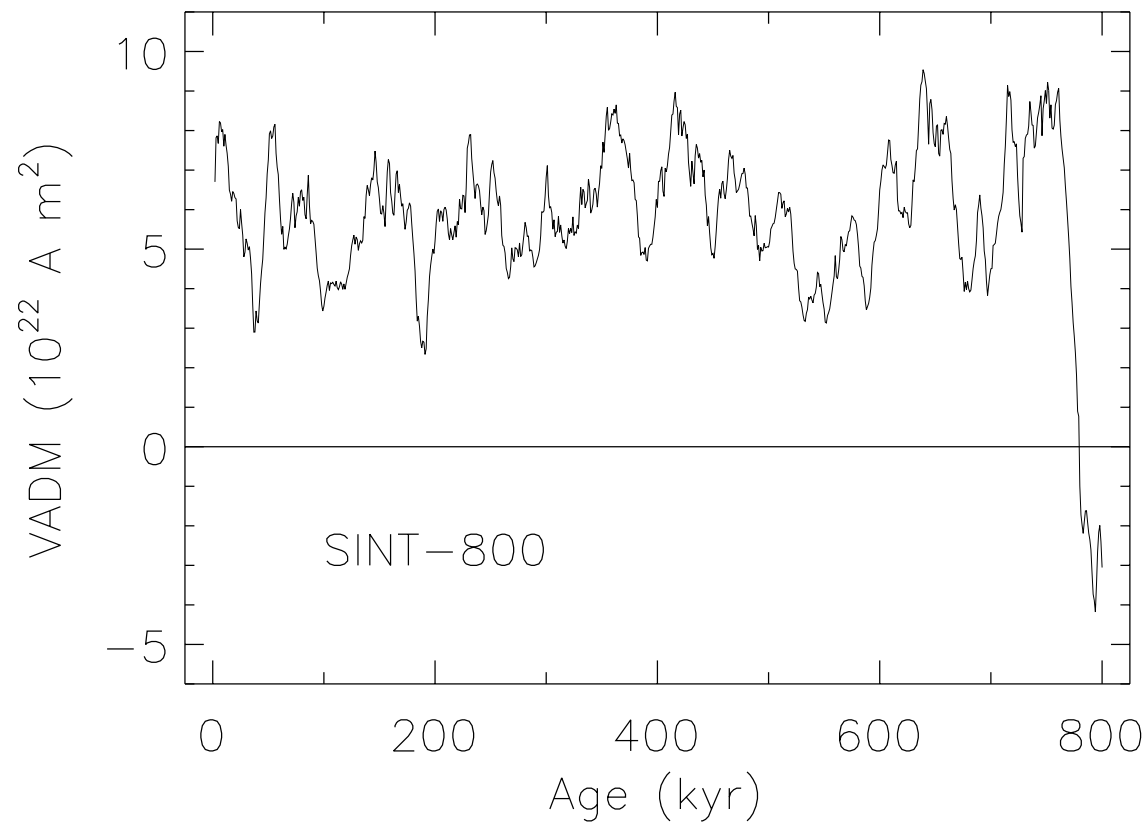
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SINT-800 VADM (Guyodo & Valet, 1999)



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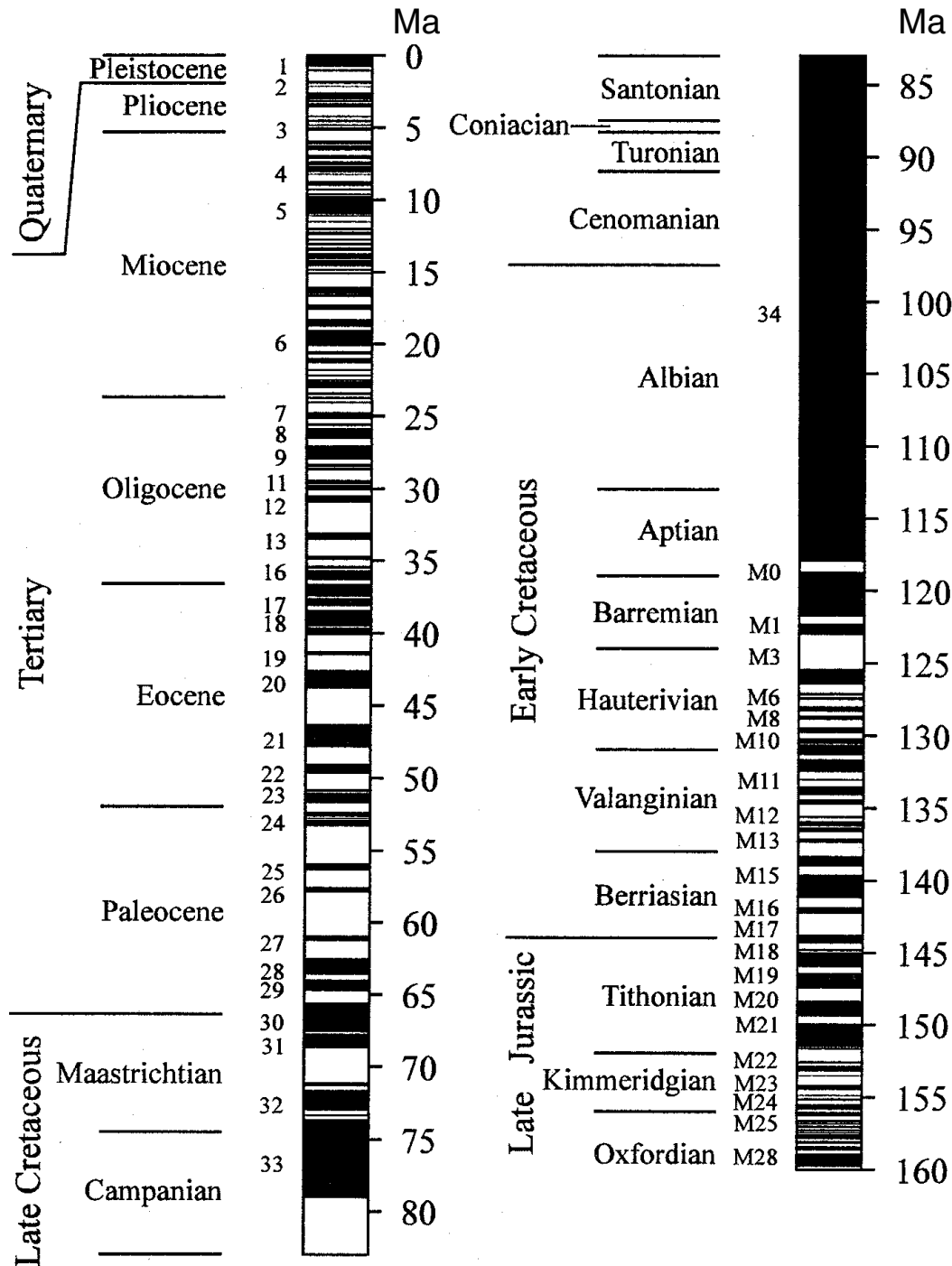
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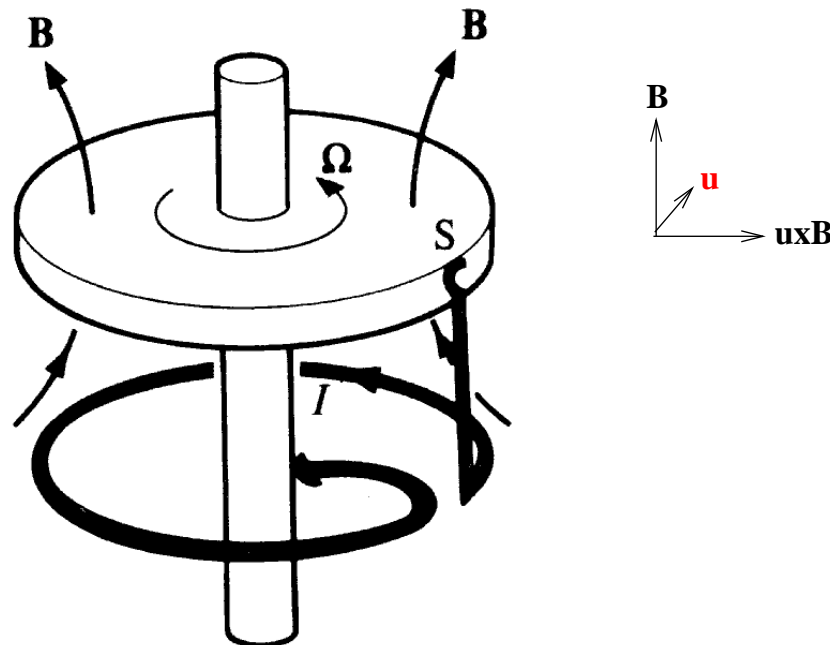
2. Dynamo theory basics

- composition of Earth
 - solid inner core, $r \approx 1200$ km, Fe, Ni
 - fluid outer core, $r \approx 3500$ km, good electrical conductor, thermal and chemical convection, location of electric currents, which excite the geomagnetic field
 - mantle, poorly conducting; thin crust
- decay time in core: $10^{4...5}$ yr
- electromagnetic induction:

$$\mathbf{u} \times \mathbf{B} \rightarrow \mathbf{j} \rightarrow \mathbf{B} \quad (\rightarrow \mathbf{u})$$

self-excitation, dynamo-electrical principle (Siemens 1867)

- disk dynamo:



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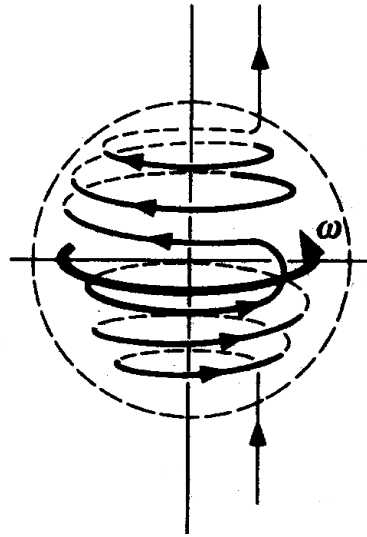
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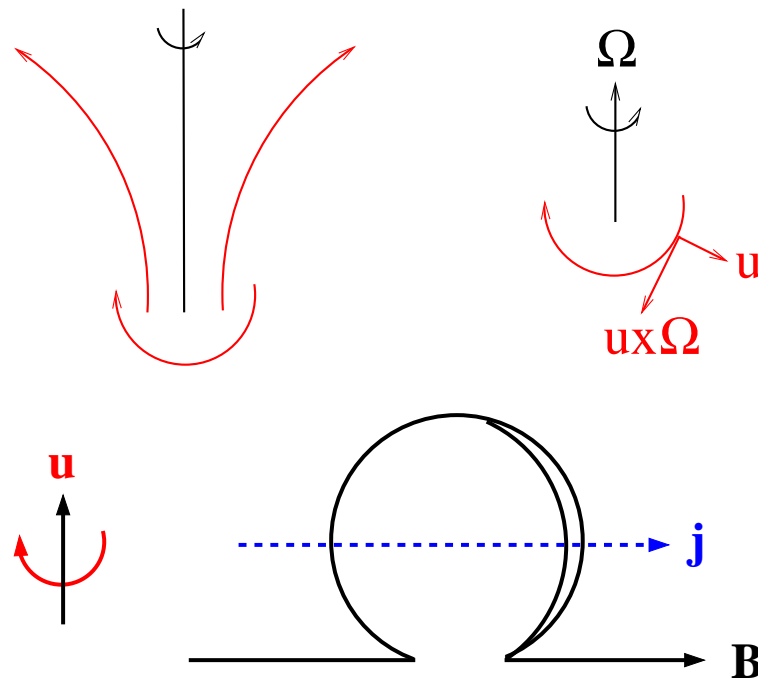
- homogenous dynamo, complicated flows, induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \text{rot}(\mathbf{u} \times \mathbf{B} - \eta \text{rot} \mathbf{B}), \quad \mathbf{B} = \mathbf{B}_{\text{pol}} + \mathbf{B}_{\text{tor}}$$

- differential rotation



- helical convection (Parker 1955)



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- theory of mean fields (Steenbeck, Krause & Rädler 1966)

$$\mathbf{B} = \bar{\mathbf{B}} + \mathbf{B}' , \quad \mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$$

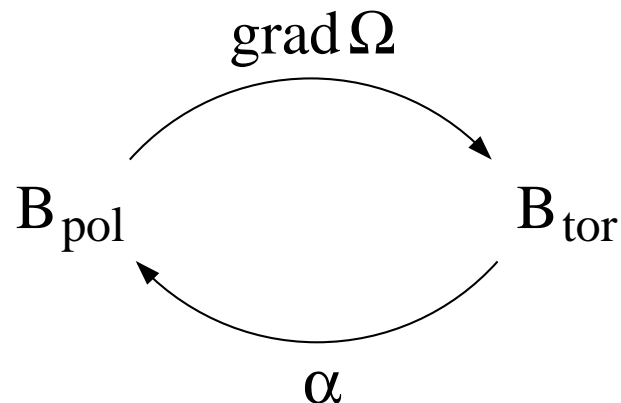
average: ensemble, azimuthal, spatial, temporal

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \text{rot}(\bar{\mathbf{u}} \times \bar{\mathbf{B}} + \overline{\mathbf{u}' \times \mathbf{B}'} - \eta \text{rot} \bar{\mathbf{B}})$$

$$\overline{\mathbf{u}' \times \mathbf{B}'} = \alpha \bar{\mathbf{B}} - \eta_T \text{rot} \bar{\mathbf{B}}$$

$$\alpha = -\frac{1}{3} \int \overline{\mathbf{u}'(t) \cdot \text{rot} \mathbf{u}'(t - \tau)} d\tau , \quad \eta_T = \frac{1}{3} \int \overline{\mathbf{u}'(t) \cdot \mathbf{u}'(t - \tau)} d\tau \gg \eta$$

- $\alpha\Omega$ -dynamo



- dynamo number $C = R_\alpha R_\Omega = \alpha_0 R / \eta_T \cdot \Omega'_0 R^3 / \eta_T$

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- stochastic helicity fluctuations $\delta\alpha$ of each convective cell

$$\alpha = \alpha_0(\theta) + \delta\alpha(\theta, t) \quad \text{with} \quad \frac{\delta\alpha}{\alpha_0} = \frac{f_{\text{rms}} F(\theta, t)}{\sqrt{2N_c \sin\theta}}$$

correlation length $R\theta_c = R\pi/N_c$, correlation time τ_c

forcing parameter $f_{\text{rms}}^2 \theta_c^2 \tau_c$

- back reaction of magnetic field on velocity (\bar{u}, u') through Lorentz force, e.g. $\alpha(B)$ -quenching
- sign of B free

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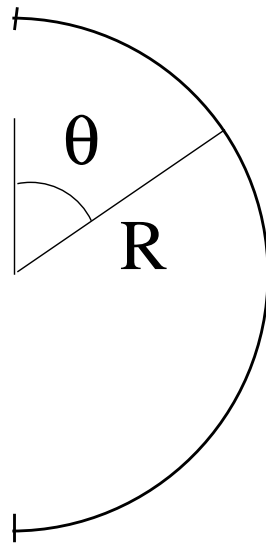
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3. 1D mean-field $\alpha\Omega$ -dynamo model

- azimuthal averages, B axisym. component of field

$$B = B(\theta, t) \exp(ikR), \quad B = \text{rot}(P \mathbf{e}_\phi) + T \mathbf{e}_\phi$$



- dimensionless $\alpha\Omega$ -dynamo equation

$$\frac{\partial}{\partial t} \begin{pmatrix} P \\ T \end{pmatrix} = \begin{pmatrix} \Delta & \alpha/\alpha_0 \\ C(\partial/\partial\theta) \sin\theta & \Delta \end{pmatrix} \begin{pmatrix} P \\ T \end{pmatrix}$$

diffusion operator Δ

dynamo number C

α fluctuations and quenching: $\alpha = \alpha_0(\theta, B) + \delta\alpha(\theta, t)$

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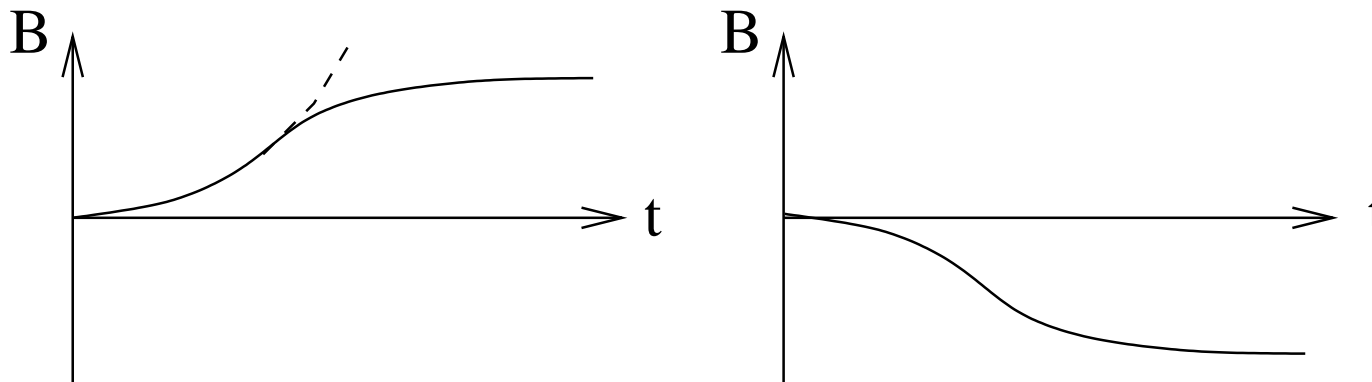
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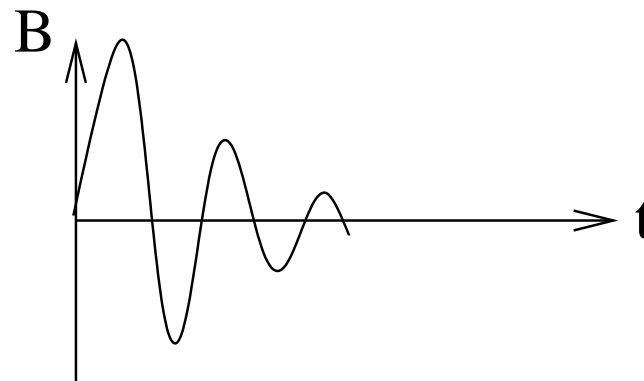
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• mode structure:

- fundamental mode: monotonous dipole, supercritical, nonlinearly saturated



- overtones: dipolar and quadrupolar, oscillatory, decaying



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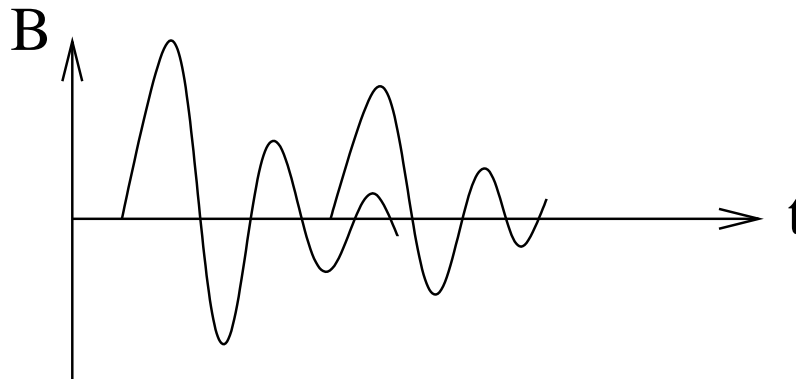
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- effect of α -fluctuations $\delta\alpha$:

→ variation of fundamental mode amplitude



→ stochastic excitation of higher modes



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4. Numerical results

- standard case: $kR = 0.5$, $C_{\text{crit}} = 57.9$, $C = 100$,
 $f_{\text{rms}} = 6.4$, $\theta_c = \pi/10$, $\tau_c = 5 \cdot 10^{-2}$

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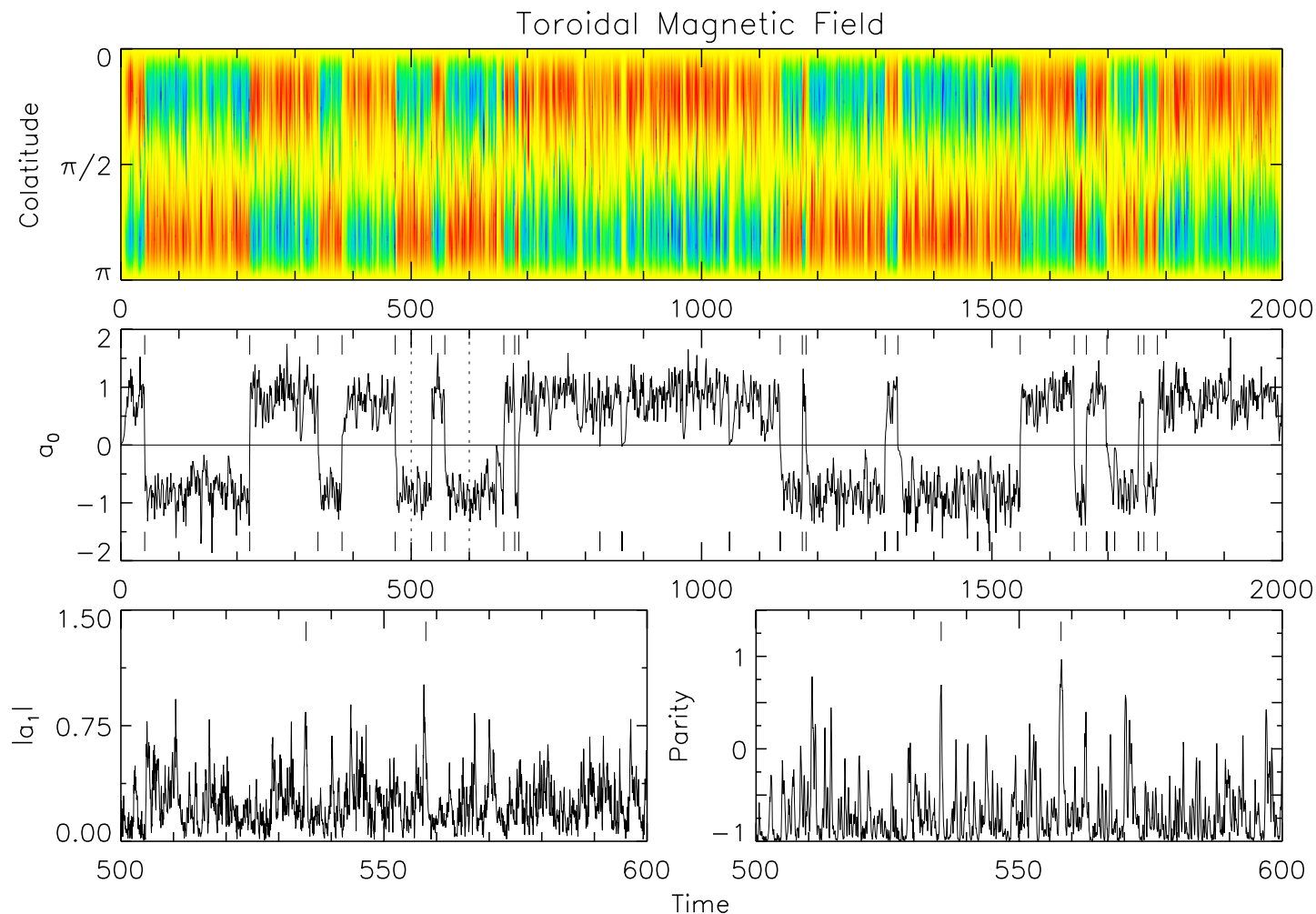
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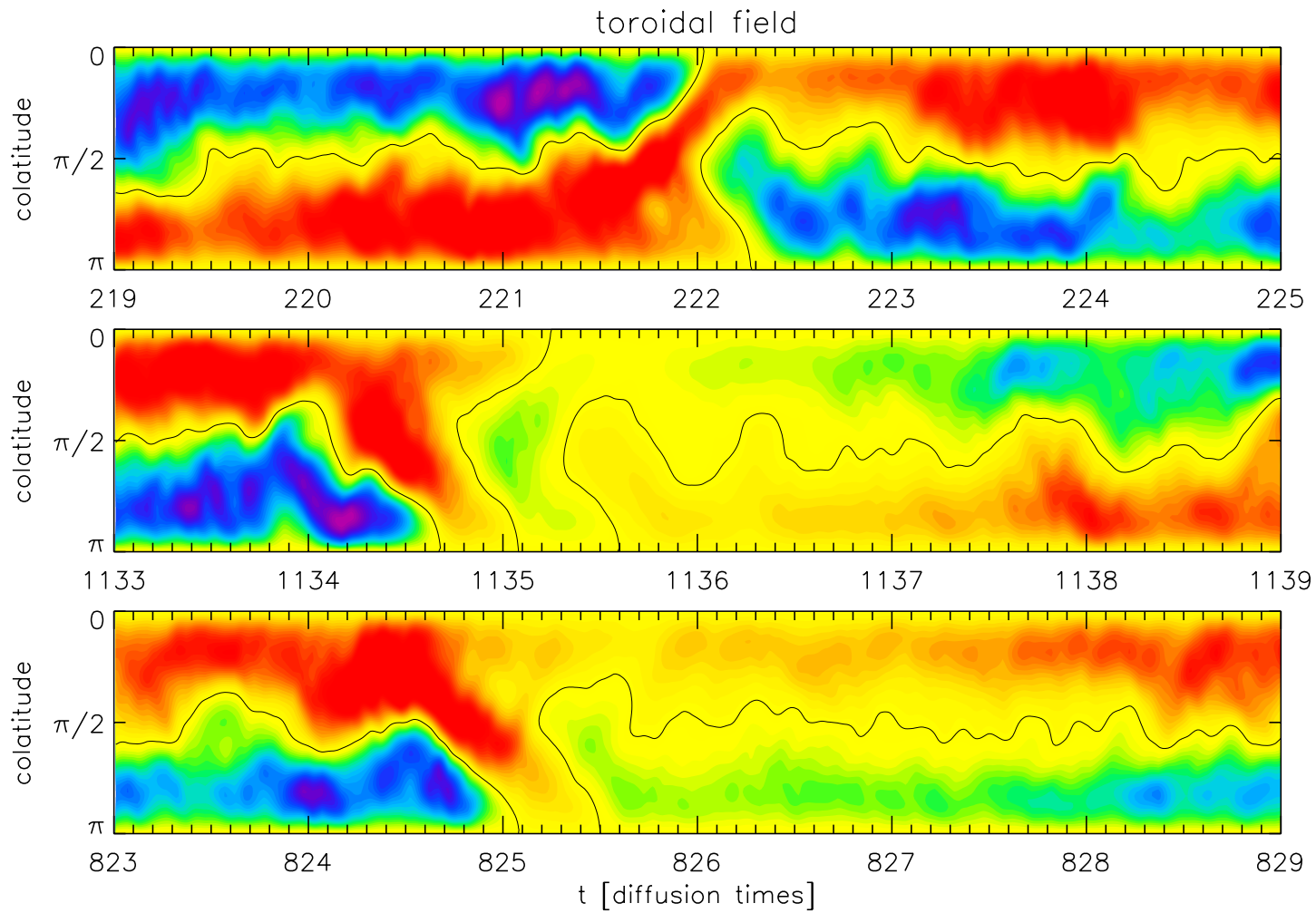


- part of much longer run with ~ 500 reversals
- secular dipole ampl. variation $dV = \langle (a - \langle a \rangle)^2 \rangle / \langle a \rangle^2 \approx 0.16$
- rapid reversals with $T_r \approx 100 \tau_D$

- reversals through decay of fundamental mode and short-time dominance of oscillating overtones

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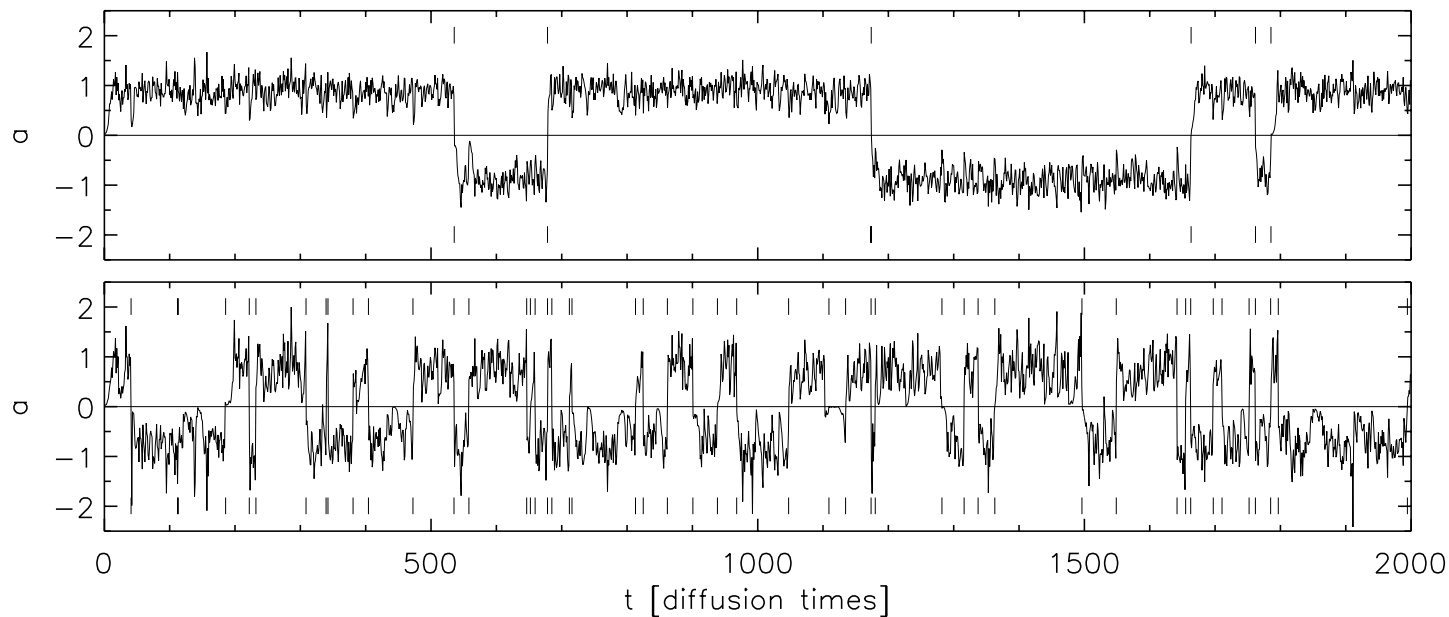
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- variation of stochastic forcing:

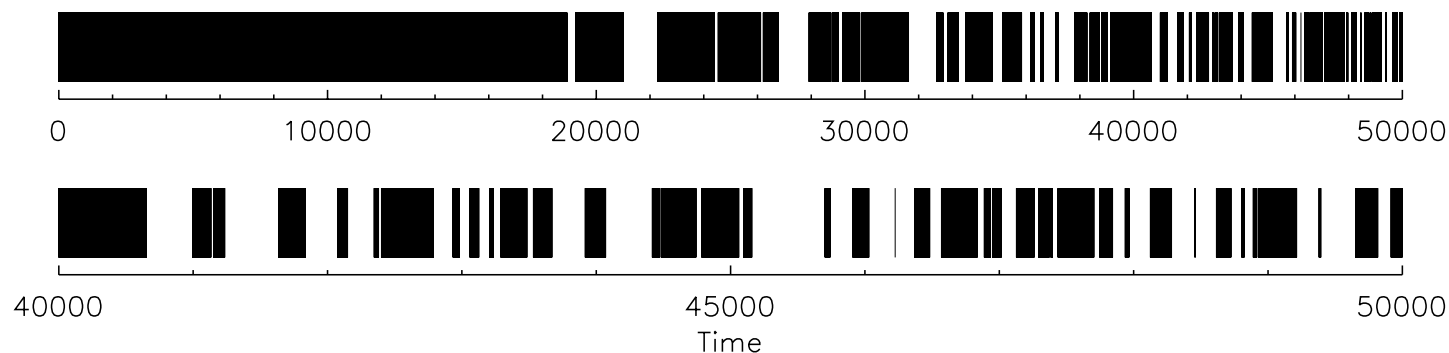
top: $f_{\text{rms}} = 5.4 \quad \leadsto \quad dv \approx 0.07, T_r \approx 500 \tau_D$

bottom: $f_{\text{rms}} = 7.4 \quad \leadsto \quad dv \approx 0.3, T_r \approx 35 \tau_D$

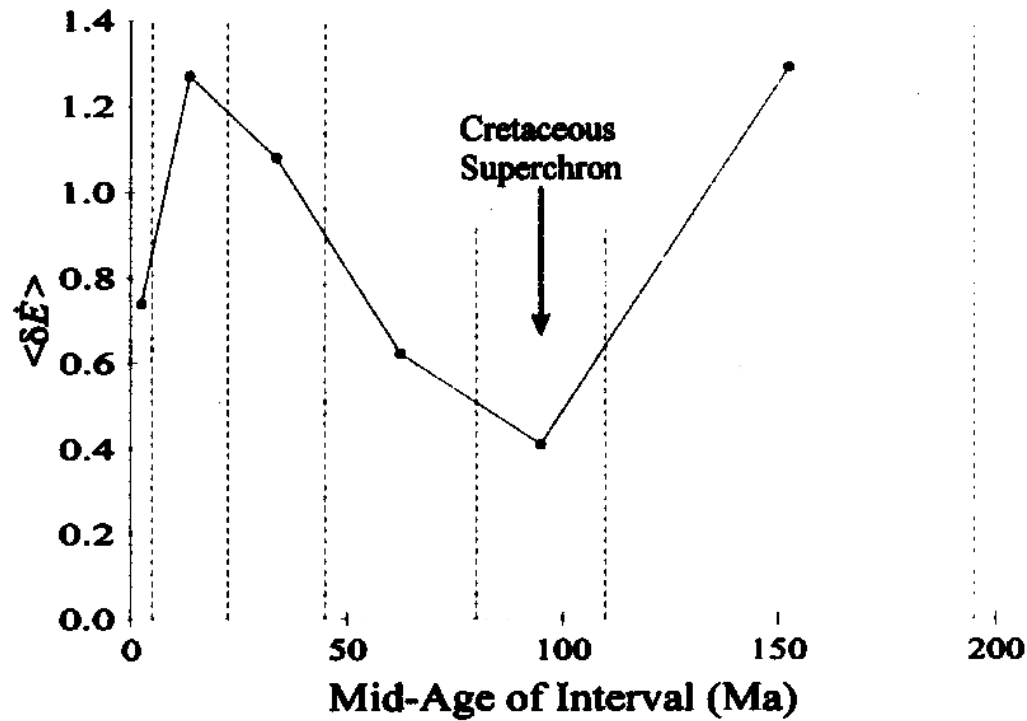


- linear increase of forcing from $f_{\text{rms}} = 4.0 \dots 6.5$

$\leadsto \quad dv \approx 0.03 \dots 0.17$ and $T_r \approx 20000 \dots 90 \tau_D$



- variability / secular variation \leftrightarrow reversal rate



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5. Theoretical analysis

- Fokker-Planck equation:

- expansion of B into unperturbed eigenmodes b_i

$$B(\theta, t) = \sum_i a_i(t) b_i(\theta)$$

$$\frac{\partial a_i}{\partial t} = \lambda_i a_i + (1 - a_0^2) \sum_k N_{ik} a_k + \sum_k F_{ik}(t) a_k$$

- averaging over fluctuations

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial a} S a + \frac{1}{2} \frac{\partial^2}{\partial a^2} D \rho$$

$\rho(a)$ probability distribution of fundamental dipole amplitude $a = a_0$

- drift term $S = \Lambda(1 - a^2)a = -dU/da$

determined by dynamo model (supercriticality, nonlinearity)

- diffusion term $D = D_0 a^2 + D_1(a)$, $D_1(0) \neq 0$

depends on forcing $f_{\text{rms}}^2 \theta_c^2 \tau_c$

variation of fundamental mode (F_{00}, D_0)

influence of overtones (F_{0k}, D_1)

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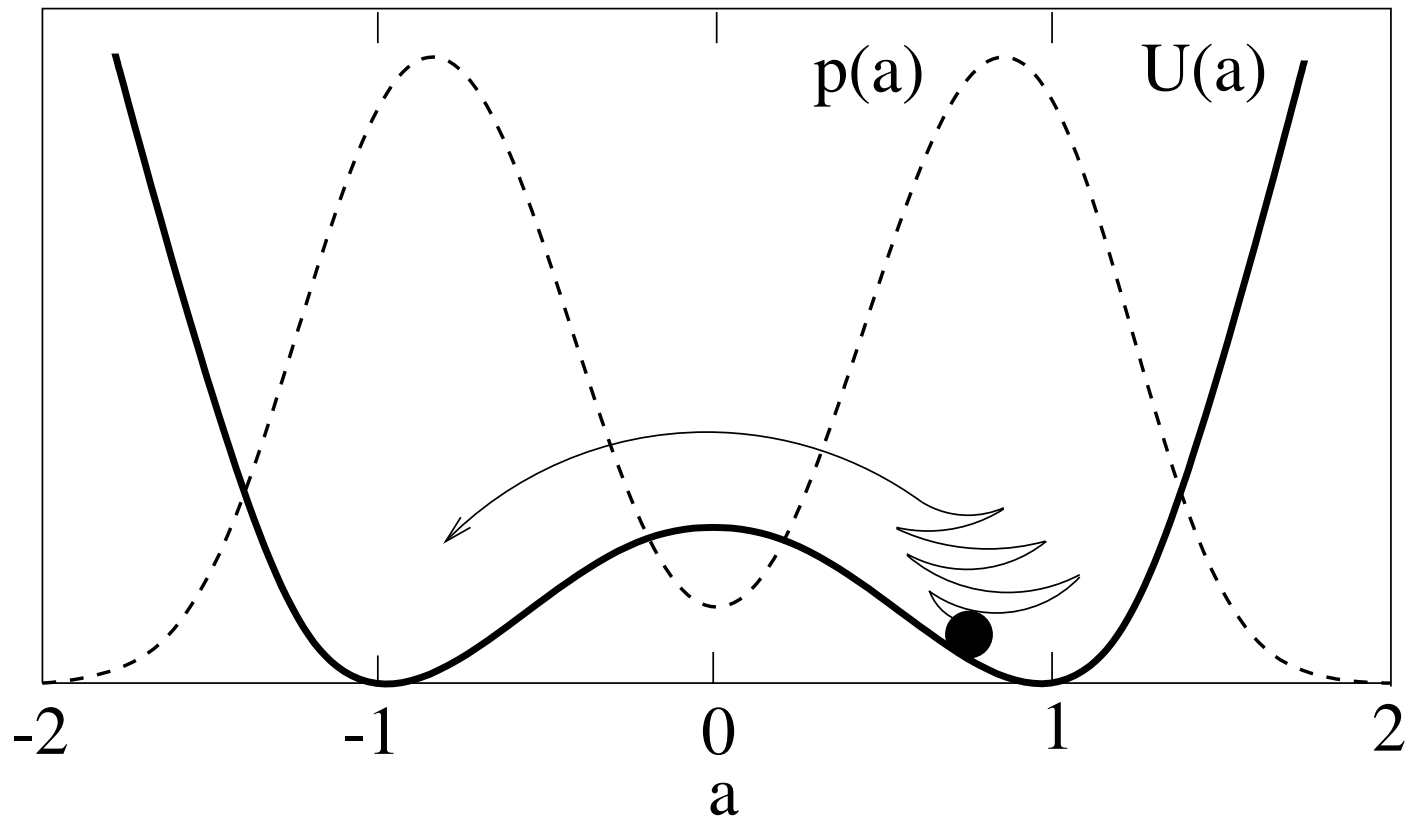
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- bistable oscillator



- $U(a)$ given by $-\partial U/\partial a = S$
- symmetry, central hill, side walls, wells
- reversals through overtones (D_1)
- reversals fast

- amplitude distribution $p(a) \propto D^{-1} \exp \int_0^a 2SD^{-1} dx \sim dv, T_r$
- $f_{\text{rms}} \nearrow \curvearrowright dv \nearrow \curvearrowright T_r \downarrow$
- ball in honey, reversal is random process, no specific cause

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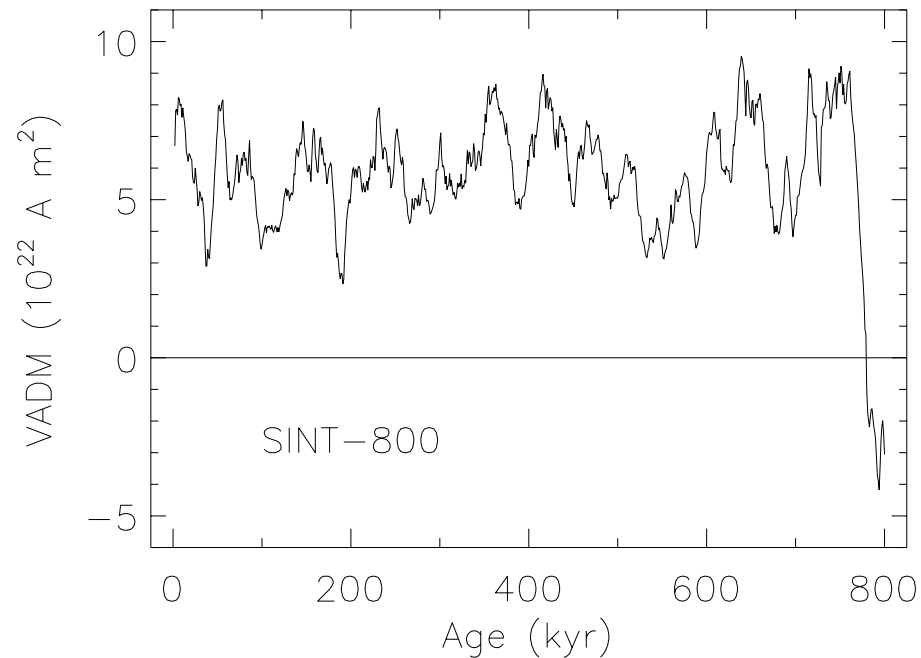
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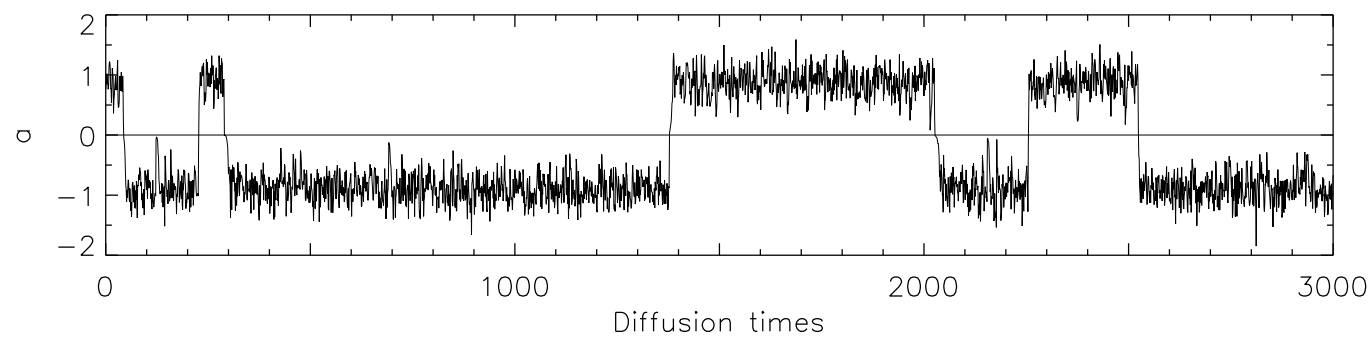
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6. Comparison with SINT-800 data

- SINT-800 VADM



- secular VADM variation $dV = 0.071 \pm 0.003$
- dynamo model $\curvearrowright T_r \approx 500\tau_D$



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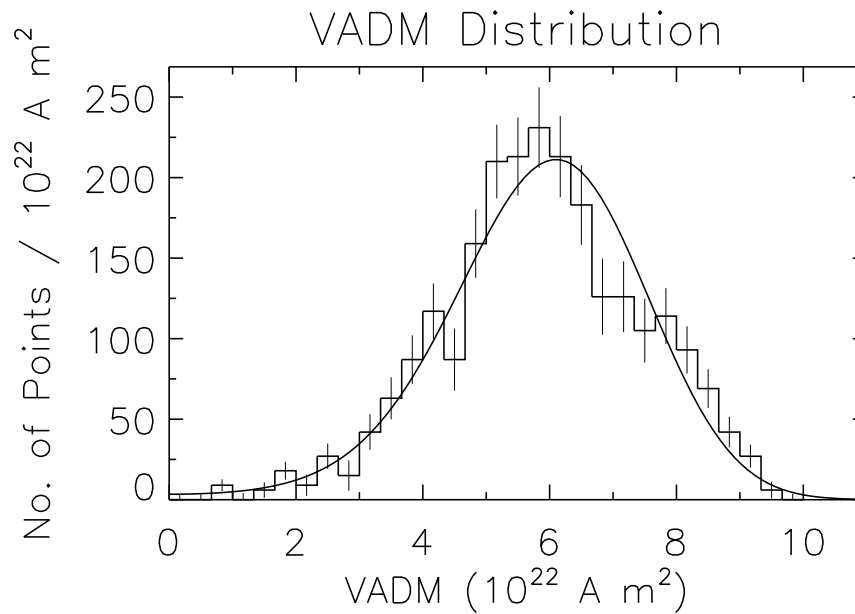


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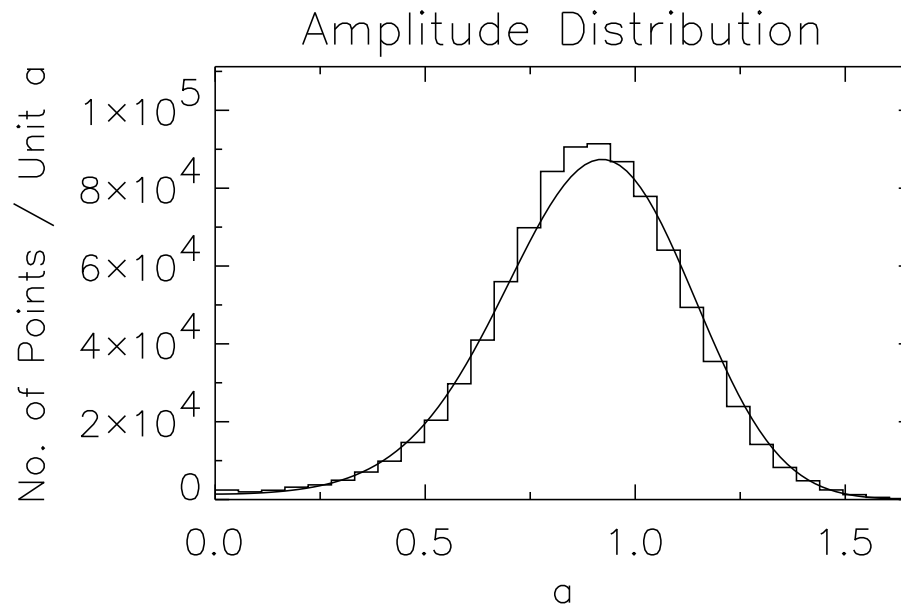
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- VADM distribution



- comparison with dynamo



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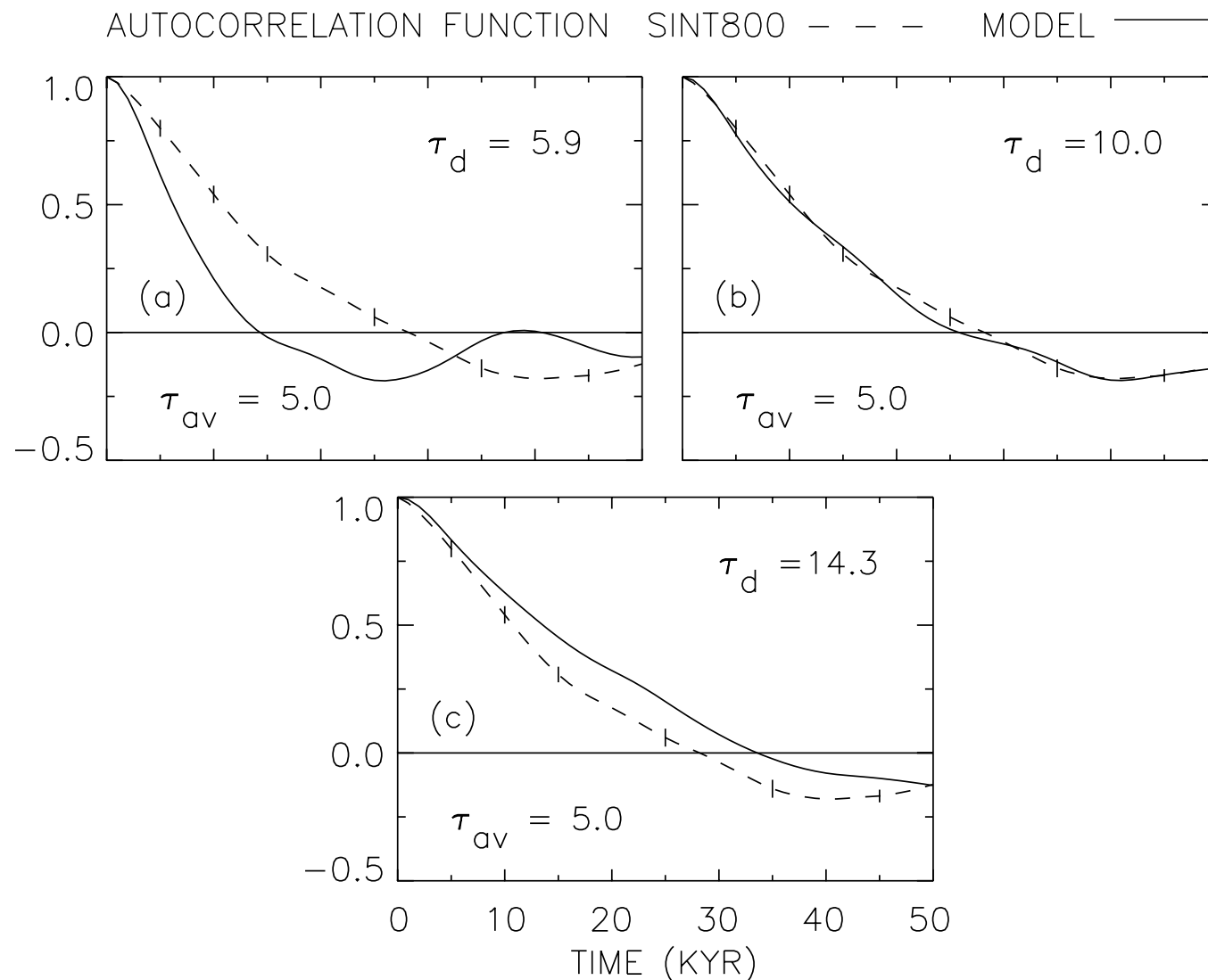
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7. Diffusion time scale τ_D

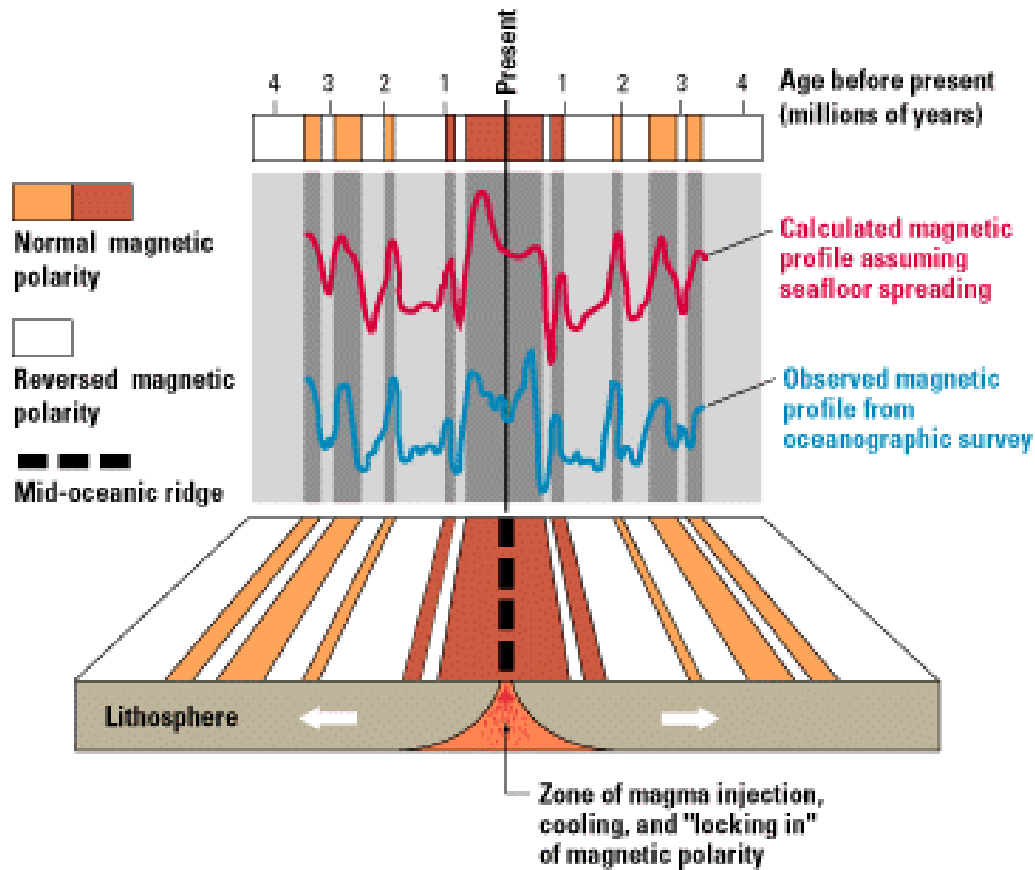
- VADM(t) and $a(t)$ are essentially the same
- absolute time \leftrightarrow turbulent diffusion time
- adjust τ_D until autocorrelation functions match



- $\tau_D \approx 10^4$ yr

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