Innerer Aufbau und Oberflächen der Planeten

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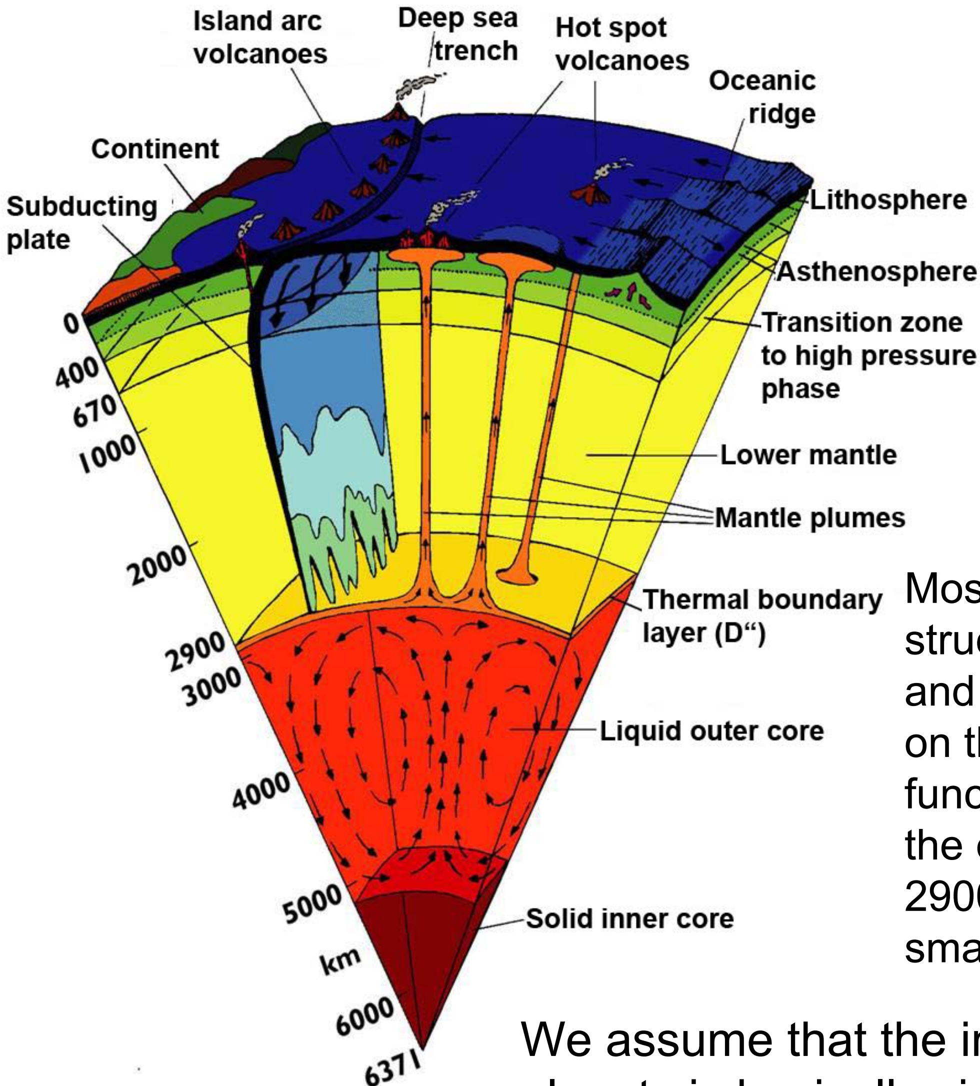
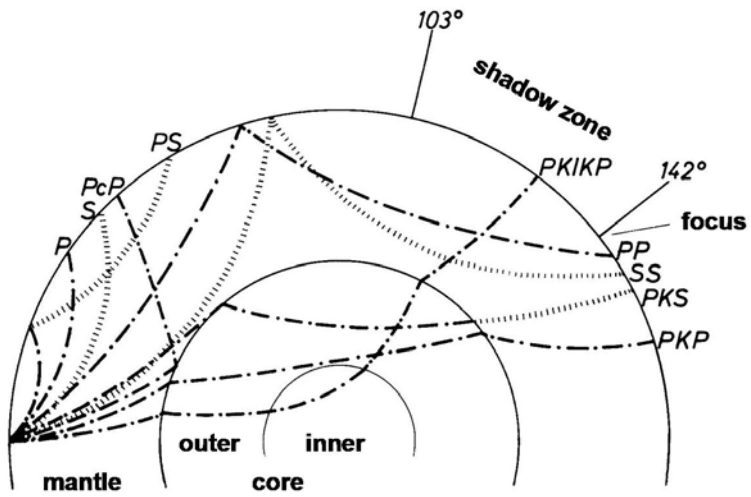
Die Erde als Prototyp eines Planeten

Informationen aus Figur, Schwerefeld und Rotation

Vergleiche terrestrischer Planeten und von Satelliten im äusseren Sonnensystem

Prototype planet Earth





Most of our knowledge on the Earth's internal structure comes from seismology (body waves and free oscillations). They provide information on the elastic properties and the density as function of radius. Basic parts of the Earth are the **crust** (6-50 km thick), the solid **mantle** (to 2900 km depth), the liquid outer **core**, and a small solid inner core.

We assume that the internal structure of other terrestrial planets is basically similar,

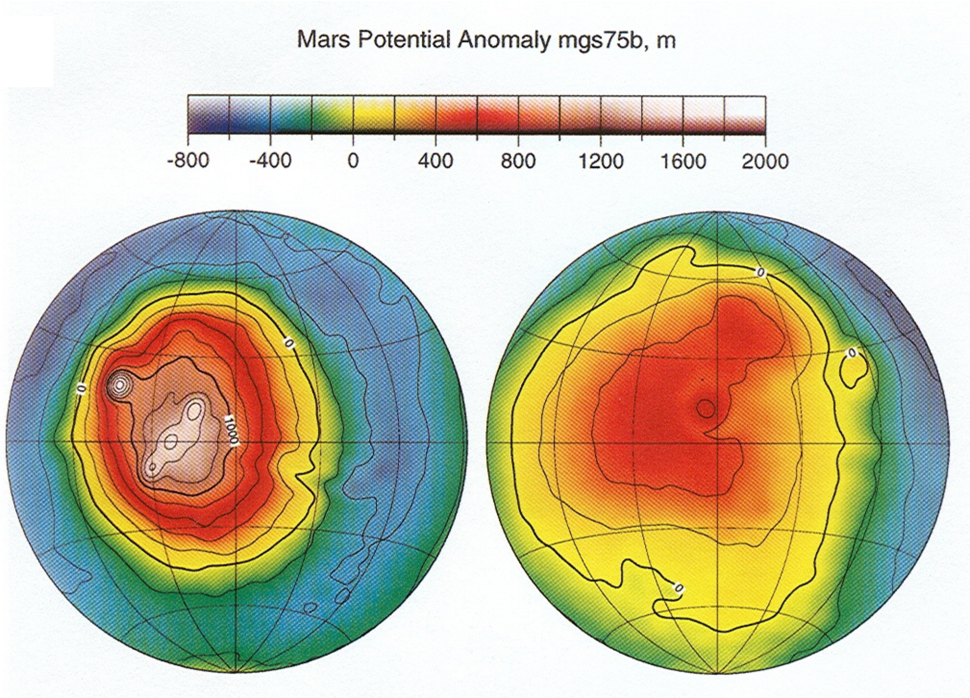
Composition of different parts of the Earth

Sources of information:
Crust – plenty of direct samples
Upper Mantle – samples from exposed mantle rock or xeno-liths (solid mantle rock carried upwards in volcanic vents)
Deep mantle and core – indirect

	Continental crust(0.2%)	Oceanic crust(0.1%)	Mantle (68%)	Core (32%)
SiO_2	60%	50%	46%	
MgO	3 %	8 %	38%	
FeO	4%	9%	8%	Fe: 85 %
Al_2O_3	17%	16%	4%	
CaO	7%	12%	3%	
Na_2O	3%	1%	<1%	
Rock type:	Granite	Basalt	Peridotite	Iron alloy
Minerals:	Quartz SiO_2 Feldspar: $CaAl_2Si_2O_8$ – $NaAlSi_3O_8$ (Plagioclase)	Plagioclase Pyroxene: $CaAlSi_2O_6$ – $(Mg,Fe)SiO_3$	Olivine: $(Mg,Fe)_2SiO_4$ Pyroxene, Garnet: $Mg_3Al_2Si_3O_{12}$	

Mantle rock is rich in magnesium (over 90% consists of Si, Mg, Fe, O). Crustal rocks contain little Mg and comparatively more Ca and Al. The core must be made mostly of iron, because Fe is the only element that (1) has the right density and compressibility at high pressure to satisfy the seismological data and (2) is sufficiently abundant. The core density is slightly less dense than iron at core pressures $\Rightarrow$ 5-10% of light elements (Si, S, O) most be present.

Information on internal structure from shape, gravity field and rotation



Seismological information is available only for the Earth and in limited amounts for the Moon. Various geodetic data put constraints on the internal structure, but the ambiguity is much larger than for seismic data.

Gravity field: fundamentals

Gravitational potential V, Gravity (acceleration) **g** = - **grad** V

Point mass: V = - GM/r (also for spherically symmetric body)

Ellipsoid: $$V = -\frac{GM}{r}\left(1 - J_2 \left(\frac{a}{r}\right)^2 P_2(\cos\theta) + \ldots\right) \qquad P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}$$

General: $$V = -\frac{GM}{r}\left(1 + \sum_{\ell=2}^{\infty} \left(\frac{a}{r}\right)^{\ell} \sum_{m=0}^{\ell} P_\ell^m(\cos\theta)\,[\,C_\ell^m \cos m\varphi + S_\ell^m \sin m\varphi\,]\right)$$

General description of gravity field in terms of spherical harmonic functions. Degree $\ell=0$ is the monopole term, $\ell=2$ the quadrupole, $\ell=3$ the octupole, etc. A dipole term does not exist when the coordinated system is fixed to the centre of mass. J, C, S are non-dimensional numbers. Note: $J_2 = -C_2^o$ (times a constant depending on the normalization of the P_ℓ^m)

Symbols [bold symbols stand for vectors]: G – gravitational constant, M –total mass of a body, r – radial distance from centre of mass, a – reference radius of planet, e.g. mean equatorial radius, θ – colatitude, φ – longitude, P_n – Legendre polynomial of degree n, P_ℓ^m – associated Legendre function of degree ℓ and order m, J_n, C_ℓ^m, S_ℓ^m – expansion coefficients

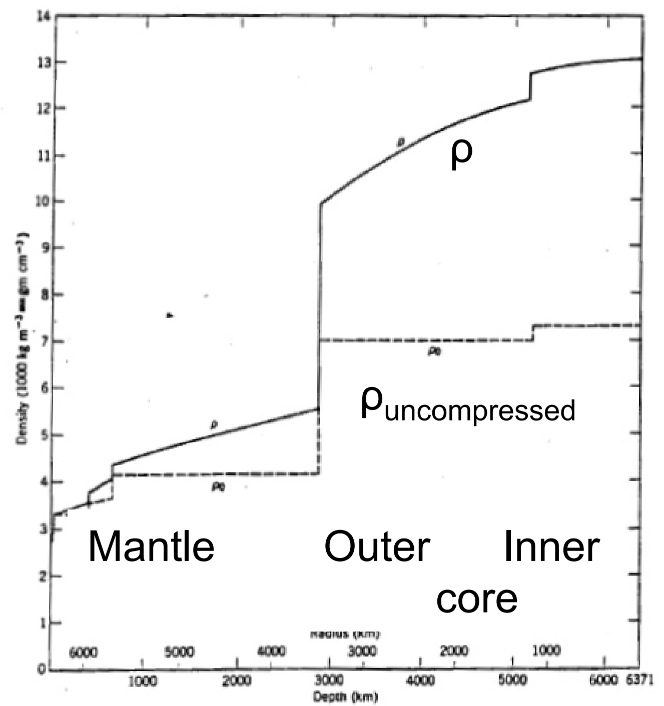
Measuring the gravity field

- Without a visiting spacecraft, the monopole gravity term (the mass M) can be determined by the orbital perturbations on other planetary bodies or from the orbital parameters of moons (if they exist)
- From a spacecraft flyby, M can be determined with great accuracy. J_2 and possibly other low-degree gravity coefficient are obtained with less accuracy
- With an orbiting spacecraft, the gravity field can be determined up to high degree (Mars up to $\ell \approx 60$, Earth up to $\ell \approx 180$)
- The acceleration of a spacecraft orbiting (or passing) another planet is determined with high accuracy by radio-doppler-tracking: The Doppler shift of the carrier frequency used for telecommunication is proportional to the line-of-sight velocity of the spacecraft relative to the receiving antenna. Δv can be measured to much better than a mm/sec.
- On Earth, direct measurements of g at many locations complement other techniques.
- The ocean surface on Earth is nearly an equipotential surface of the gravity potential. Its precise determination by laser altimetry from an orbiting S/C reveals small-wavelength structures in the gravity field.

Mean density and uncompressed density

From the shape (volume) and mass of a planet, the mean density ρ_{mean} is obtained. It depends on chemical composition, but through self-compression also on the size of the planet (and its internal temperature; in case of terrestrial planets only weakly so). In order to compare planets of different size in terms of possible differences in composition, an **uncompressed density** can be calculated - the mean density it would have, when at its material where at 1 bar. This requires knowledge of incompressibility k (from high-pressure experiments or from seismology in case of the Earth).

	ρ_{mean}	$\rho_{uncompressed}$	
Earth	5515	4060	kg m^{-3}
Moon	3341	3315	kg m^{-3}



The mean density alone gives no clue on the radial distribution of density: a body could be an undifferentiated mixture (e.g. of metal and silicate, or of ice and rock in the outer solar system), or could have separated in different layers (e.g. mantle and core).

Moment of inertia

$\mathbf{L} = \mathbf{I}\omega$ L: Angular momentum, ω: angular frequency, I moment of inertia

$$I = \iiint \rho\, s^2 dV$$ for rotation around an arbitrary axis, s is distance from that axis

I is a symmetric tensor. It has 3 principal axes and 3 principal components (maximum, intermediate, minimum moment of inertia: C ≥ B ≥ A.) For a spherically symmetric body rotating around polar axis

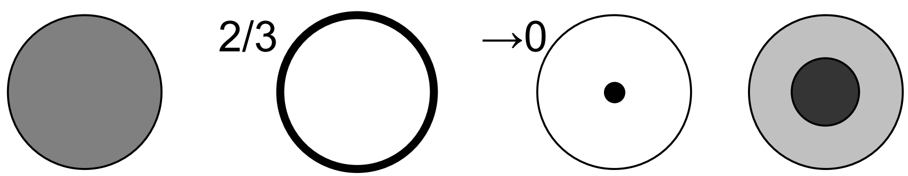
$$C = \frac{8\pi}{3}\int_0^a \rho(r)\, r^4 dr$$

compare with integral for mass

$$M = 4\pi \int_0^a \rho(r)\, r^2 dr$$

In planetary science, the maximum moment of a nearly radially symmetric body is usually expressed as $C/(Ma^2)$, a dimensionless number. Its value provides information on how strongly the mass is concentrated towards the centre.

$C/(Ma^2)=0.4$ 2/3 →0



0.347 for $a_c=a/2$, $\rho_c=2\rho_m$
0.241 for $a_c=a/2$, $\rho_c=10\rho_m$

Homogeneous sphere | Hollow shell | Small dense core thin envelope | Core and mantle, each with constant density

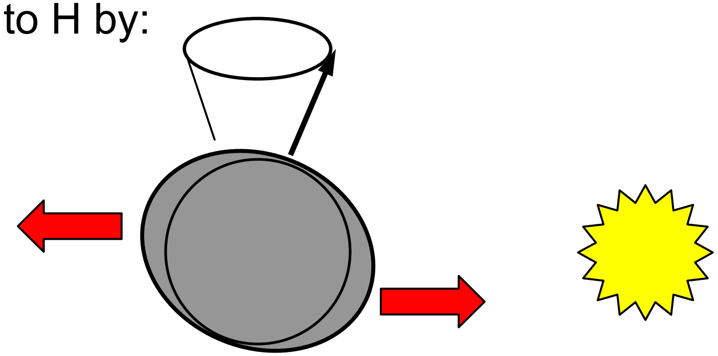
Symbols: L – angular momentum, I moment of inertia (C,B,A – principal components), ω rotation frequency, s – distance from rotation axis, dV – volume element, M – total mass, a – planetary radius (reference value), a_c – core radius, ρ_m – mantle density, ρ_c –core density

Determining planetary moments of inertia

McCullagh's formula $J_2 = \frac{C - \frac{1}{2}(B+A)}{Ma^2}$ for ellipsoid (B=A): $J_2 = \frac{C-A}{Ma^2}$

In order to obtain $C/(Ma^2)$, the dynamical ellipticity H = (C-A)/C is needed. It can be uniquely determined from observation of the precession of the planetary rotation axis due to the solar torque (plus lunar torque in case of Earth) on the equatorial bulge. For solar torque alone, the precession frequency relates to H by:

$$\omega_P = \frac{3}{2} \frac{\omega_{orbit}^2}{\omega_{spin}} H \cos \varepsilon$$



When the body is in a locked rotational state (Moon), H can be deduced from nutation.

For the Earth $T_P = 2\pi/\omega_P = 25{,}800$ yr (but here also the lunar torque must be accounted)

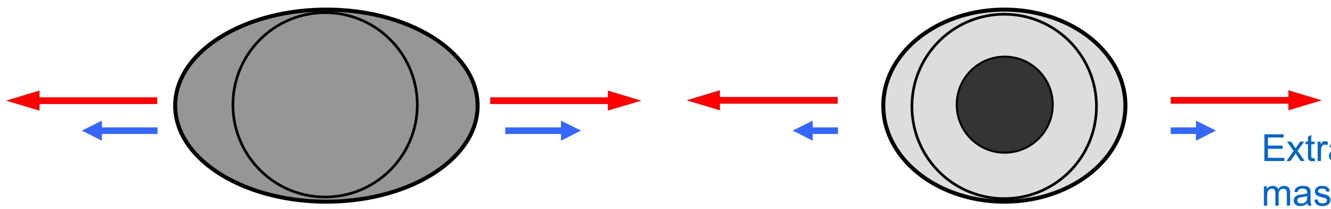
$H = 1/306$ and $J_2 = 1.08\times10^{-3}$ $\Rightarrow$ $C/(Ma^2) = J_2/H = 0.3308$.

This value is used, together with free oscillation data, to constrain the radial density distribution.

Symbols: J_2 – gravity moment, ω_P precession frequency, ω_{orbit} – orbital frequency (motion around sun), ω_{spin} – spin frequency, ε - obliquity

Determining planetary moments of inertia II

For many bodies no precession data are available. If the body rotates sufficiently rapidly and if its shape can assumed to be in hydrostatic equilibrium [i.e. equipotential surfaces are also surfaces of constant density], it is possible to derive $C/(Ma^2)$ from the degree of ellipsoidal flattening or the effect of this flattening on the gravity field (its J_2-term). At the same spin rate, a body will flatten less when its mass is concentrated towards the centre.



Darwin-Radau theory for an slightly flattened ellipsoid in hydrostatic equilibrium

$$\mathrm{m} = \frac{\omega_{\mathrm{spin}}^2 \mathrm{a}^3}{\mathrm{GM}}$$ measures rotational effects (ratio of centrifugal to gravity force at equator).

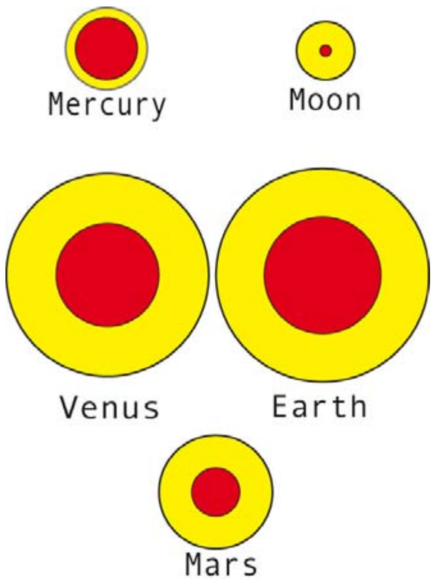
Flattening is f = (a-c)/a. The following relations hold approximately:

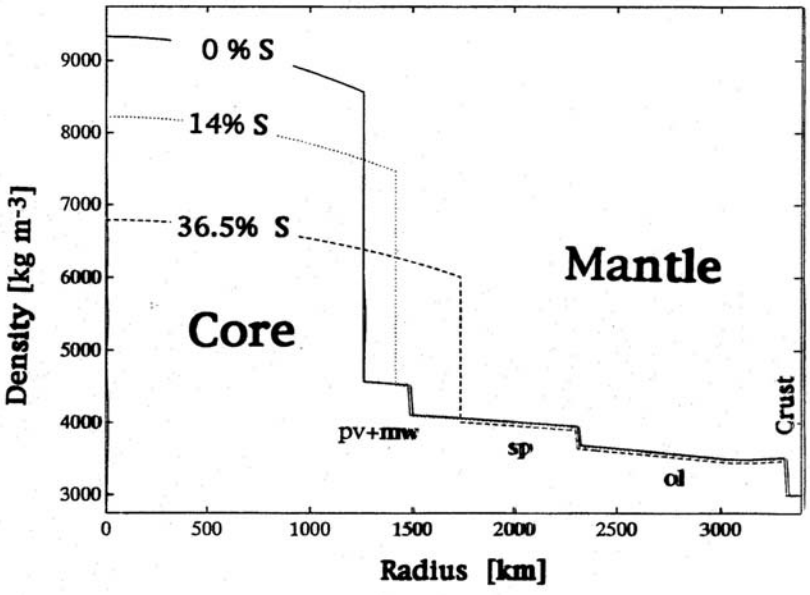
$$f = \frac{3}{2} J_2 + \frac{1}{2} m \qquad \frac{C}{Ma^2} = \frac{2}{3} - \frac{4}{15}\sqrt{\frac{5}{2}\frac{m}{f} - 1} \qquad \frac{C}{Ma^2} = \frac{2}{3} - \frac{4}{15}\sqrt{\frac{4m - 3J_2}{m + 3J_2}}$$

Symbols: a –equator radius, c- polar radius, f – flattening, m – centrifugal factor (non-dimensional number)

Structural models for terrestrial planets

	ρ_{mean} kg m^{-3}	$\rho_{uncompr}$ kg m^{-3}	C/Ma^2
Mercury	5430	5280	?
Venus	5245	3990	?
Earth	5515	4060	0.3308
Moon	3341	3315	0.390
Mars	3935	3730	0.366



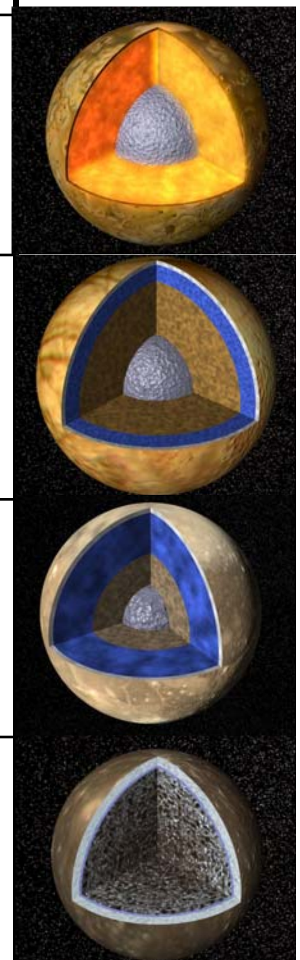


Assuming that the terrestrial planets are made up of the same basic components as Earth (silicates / iron alloy with zero-pressure densities of 3300 kg m^{-3} and 7000 kg m^{-3}, respectively), core sizes can be derived.

Ambiguities remain, even when ρ_{mean} and C/Ma^2 are known: the three density models for Mars satisfy both data, but have different core radii and densities with different sulphur contents in the core.

Interior of Galilean satellites

	ρ [kg m^{-3}]	C/Ma^2
Io Silicate mantle Iron core	3530	0.378
Europa (Thin) ice layer Silicate mantle Iron core	3020	0.347
Ganymede Thick ice shell Silicate mantle Iron core	1940	0.311
Callisto Ice layer Ice/silicate/iron mixture below	1850	0.358



From close Galileo flybys mean density and J_2 (assume hydrostatic shape $\Rightarrow C/(Ma^2)$)

Low density of outer satellites $\Rightarrow$ substantial ice (H_2O) component.

Three-layer models (ice, rock, iron) except for Io. Assume rock/Fe ratio.

Callisto's C/Ma^2 too large for complete differentiation $\Rightarrow$ core is probably an undifferentiated rock-ice mixture.