Space plasma physics

- Basic plasma properties and equations
- Space plasmas, examples and phenomenology
- Single particle motion and trapped particles
- Collisions and transport phenomena
- Elements of kinetic theory
- Fluid equations and magnetohydrodynamics
- Magnetohydrodynamic waves

Space plasma physics

- Boundaries, shocks and discontinuities
- Plasma waves in the fluid picture I
- Plasma waves in the fluid picture II
- Fundamentals of wave kinetic theory
- Concepts of plasma micro- and macroinstability
- Kinetic plasma microinstabilities
- Wave-particle interactions

Basic plasma properties and equations

- **Definition of a plasma**
- **Space plasmas - phenomenology**
- **Parameters**
- **Currents and charge densities**
- **Composition and ionization**
- **Maxwell‘s equations and forces**
- **Induction equation**

Definition of a plasma

A plasma is a mixed gas or fluid of neutral and charged particles. Partially or fully ionized space plasmas have usually the same total number of positive (ions) and negative (electrons) charges and therefore behave quasineutral.

Space plasma particles are mostly free in the sense that their kinetic exceeds their potential energy, i.e. they are normally hot, $T > 1000$ K.

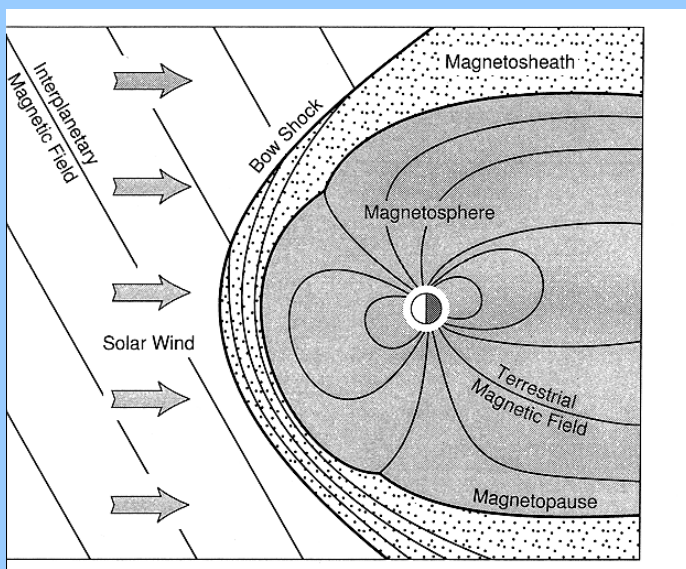
Space plasmas have typically ***vast dimensions***, such that the free paths of thermal particles are larger than the typical spatial scales --> they are ***collisionless***.

Different types plasmas

Plasmas differ by their chemical composition and the ionization degree of the ions or molecules (from different sources). Plasmas are mostly magnetized (internal and external magnetic fields).

- Solar interior and atmosphere
- Solar corona and wind (heliosphere)
- Planetary magnetospheres (plasma from solar wind)
- Planetary ionospheres (plasma from atmosphere)
- Coma and tail of a comet
- Dusty plasmas in planetary rings

Schematic topography of solar-terrestrial environment



solar wind -> magnetosphere -> iononosphere

Different plasma states

Plasmas differ by the charge, e_j, mass, m_j, temperature, T_j, density, n_j, bulk speed U_j and thermal speed, $V_j=(k_B T_j/m_j)^{1/2}$ of the particles (of species j) by which they are composed.

- Long-range (shielded) Coulomb potential
- Collective behaviour of particles
- Self-consistent electromagnetic fields
- Energy-dependent (often weak) collisions
- Reaction kinetics (ionization, recombination)
- Variable sources (pick-up)

Debye shielding

The mobility of free electrons leads to shielding of point charges (dressed particles) and their Coulomb potential.

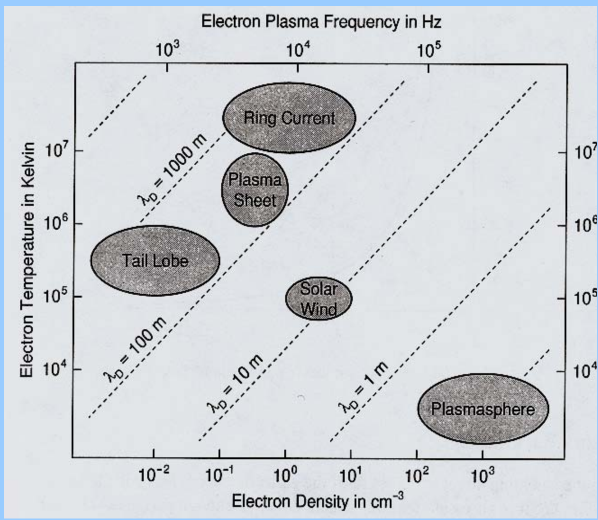
$$\phi_D = \frac{q}{4\pi\epsilon_0 r} exp\left(-\frac{r}{\lambda_D}\right)$$

The exponential function cuts off the electrostatic potential at distances larger than Debye length, λ_D, which for $n_e = n_i$ and $T_e = T_i$ is:

$$\lambda_D = \left(\frac{\epsilon_0 k_B T_e}{n_e e^2}\right)^{1/2}$$

The plasma is ***quasineutral*** on large scales, $L >> \lambda_D$, otherwise the shielding is ineffective, and one has microscopically a simple ionized gas. The plasma parameter (number of particles in the Debye sphere) must obey, $\Lambda = n_e \lambda_D^3 >> 1$, for ***collective behaviour*** to prevail.

Ranges of electron density and temperature for geophysical plasmas



Some plasmas, like the Sun's chromosphere or Earth's ionosphere are not fully ionized. Collisions between neutrals and charged particles couple the particles together, with a typical collision time, τ_n, say. Behaviour of a gas or fluid as a plasma requires that:

$$\omega_{pe}\tau_n >> 1$$

Specific plasma parameters

Coulomb force -> space charge oscillations

Lorentz force -> gyration about magnetic field

Any perturbation of quasineutrality will lead to electric fields accelerating the light and mobile electrons, thus resulting in fast collective motions -> ***plasma oscillations*** around the inert and massive ions at the plasma frequency:

$$\omega_{pe} = \left(\frac{n_e e^2}{m_e \epsilon_0}\right)^{1/2}$$

The Lorentz force acts perpendicularly to the magnetic field and bends the particle motion, thus leading to circulation (electrons in clockwise, and ions in anti-clockwise sense) about the field -> ***gyromotion*** at the gyro- or cyclotron frequency:

$$\omega_g = \frac{qB}{m}$$

Theoretical descriptions of a plasma

Plasma dynamics is governed by the interaction of the charged particles with the self-generated (by their motions through their charge and current densities) electromagnetic fields. These internal fields feed back onto the particles and make plasma physics difficult.

- Single particle motion (under external fields)
- Magnetohydrodynamics (single fluid and Maxwell‘s equations)
- Multi-fluid approach (each species as a separate fluid)
- Kinetic theory (Vlasov-Boltzmann description in terms of particle velocity distribution functions and field spectra)

Electromagnetic field (Maxwell‘s) equations

The motion of charged particles in space is strongly influenced by the self-generated electromagnetic fields, which evolve according to ***Ampere‘s and Faraday‘s*** (induction) laws:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

where ε_0 and μ_0 are the vacuum dielectric constant and free-space magnetic permeability, respectively. The charge density is ρ and the current density **j**. The electric field obeys ***Gauss*** law and the magnetic field is always free of divergence, i.e. we have:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

Electromagnetic forces and charge conservation

The motion of charged particles in space is determined by the electrostatic ***Coulomb*** force and magnetic ***Lorentz*** force:

$$\mathbf{F}_C = q\mathbf{E} \qquad \mathbf{F}_L = q(\mathbf{v} \times \mathbf{B})$$

where q is the charge and **v** the velocity of any charged particle. If we deal with electrons and various ionic species (index, s), the ***charge and current densities*** are obtained, respectively, by summation over all kinds of species as follows:

$$\rho = \sum_s q_s n_s \qquad \mathbf{j} = \sum_s q_s n_s \mathbf{v}_s$$

which together obey the ***continuity equation***, because the number of charges is conserved, i.e. we have:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Lorentz transformation of the electromagnetic fields

Let S be an inertial frame of reference and S' be another frame moving relative to S at constant velovity **V**. Then the electromagnetic fields in both frames are connected to each other by the ***Lorentz transformation***:

$$\begin{aligned} E'_x &= E_x \\ E'_y &= \gamma(E_y - VB_z) \\ E'_z &= \gamma(E_z + VB_y) \end{aligned}$$

$$\begin{aligned} B'_x &= B_x \\ B'_y &= \gamma\left(B_y + \frac{V}{c^2}E_z\right) \\ B'_z &= \gamma\left(B_z - \frac{V}{c^2}E_y\right) \end{aligned}$$

where $\gamma = (1-V^2/c^2)^{-1/2}$ is the ***Lorentz factor*** and c the speed of light. In the non-relativistic case, $V << c$, we have $\gamma = 1$, and thus $\mathbf{B}' \approx \mathbf{B}$. The magnetic field remains to lowest order unchanged in frame transformations.

However, the electric field obeys, $\mathbf{E}' \approx \mathbf{E} + \mathbf{V} \times \mathbf{B}$. A space plasma is usually a very good conductor, and thus we have, $\mathbf{E}' = \mathbf{0}$, and the result, $\mathbf{E} \approx - \mathbf{V} \times \mathbf{B}$, which is called the ***convection electric field***.

Induction equation

In order to study the transport of plasma and magnetic field lines quantitatively, let us consider the fundamental induction equation, i.e. ***Faraday's law*** in combination with the simple phenomenological ***Ohm's law***, relating the electric field in the plasma frame with its current:

$$\mathbf{j} = \sigma_0(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Using ***Ampere's law*** for slow time variations, without the displacement current and the fact that the field is free of divergence ($\nabla \cdot \mathbf{B} = 0$), yields the induction equation (with conductivity σ_0):

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma_0} \nabla^2 \mathbf{B}$$

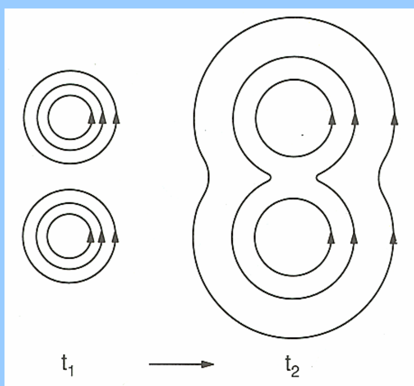
Convection   Diffusion

Magnetic diffusion

Assuming the plasma be at rest, the induction equation becomes a pure diffusion equation:

$$\frac{\partial \mathbf{B}}{\partial t} = D_m \nabla^2 \mathbf{B}$$

with the magnetic ***diffusion coefficient*** $D_m = (\mu_0 \sigma_0)^{-1}$.

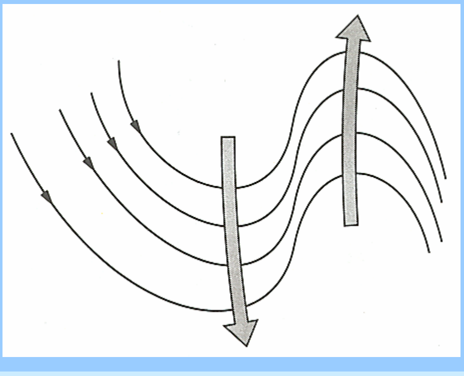


Under the influence of finite resistivity the magnetic field diffuses across the plasma, and field inhomogenities are smoothed out at time scale, $\tau_d = \mu_0 \sigma_0 L_B^2$, with scale length L_B.

Hydromagnetic theorem

In an ideal collisionless plasma in motion with infinite conductivity the induction equation becomes:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$



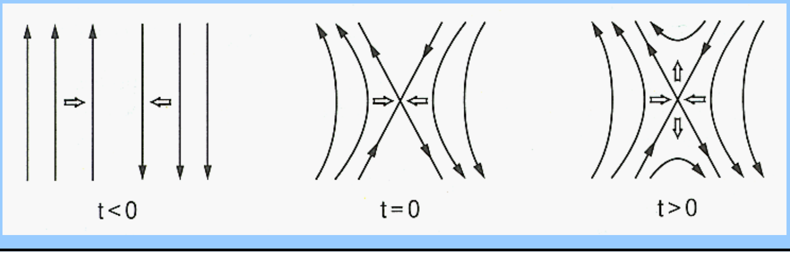
The field lines are constrained to move with the plasma -> ***frozen-in field***. If plasma patches on different sections of a bundle of field lines move oppositely, then the lines will be deformed accordingly. Electric field in plasma frame, **E**' = **0**, **->** voltage drop around closed loop is zero**.**

Magnetic merging - reconnection

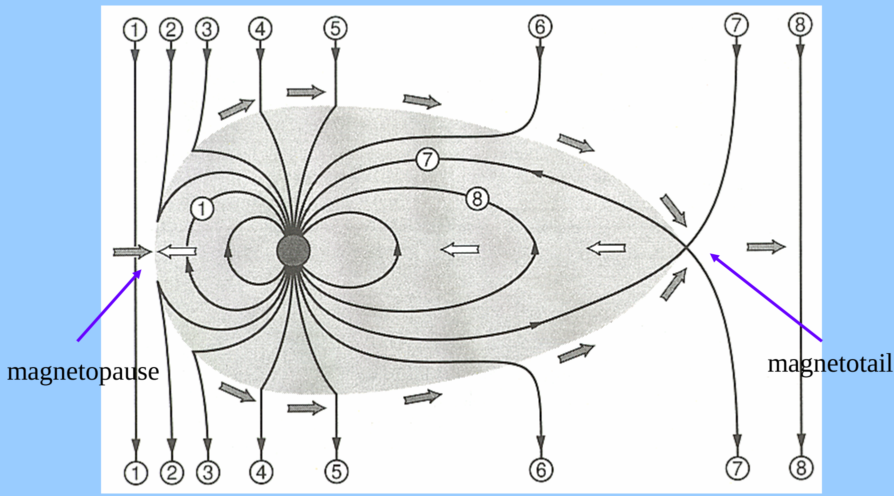
Assuming the plasma streams at bulk speed *V*, then the induction equation can be written in simple dimensional form as:

$$\frac{B}{\tau} = \frac{VB}{L_B} + \frac{B}{\tau_d}$$

The ratio of the first to second term gives the so-called ***magnetic Reynolds number***, $R_m = \mu_0 \sigma_0 L_B V$, which is useful to decide whether a plasma is diffusion or convection dominated. Current sheet with converging flows -> magnetic merging at points where $R_m \approx 1$. Field lines form *X-point* and *separatrix.*



Field line merging and reconnection in the Earth's magnetosphere



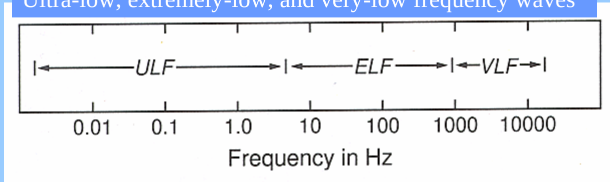
Waves in plasmas I

In a plasma there are many reasons for spatio-temporal variations (waves or more generally ***fluctuations***): High temperature required for ***ionization*** ($\Phi_H = 13.6$ eV $\approx$ 158000 K) implies fast *thermal particle motion*. As a consequence

-> microscopic fluctuating charge separations and currents
-> fluctuating electromagnetic fields.

There are also ***externally imposed disturbances*** which may propagate through the plasma and spread their energy in the whole plasma volume. The relevant frequency ranges are:

Ultra-low, extremely-low, and very-low frequency waves



Waves in plasmas II

Plasma waves are not generated at random. To exist they must satisfy two conditions:

-> their amplitude must exceed the thermal noise level
-> they must obey appropriate dynamic plasma equations

There is a large variety of ***wave modes*** which can be excited in a plasma. The mode structure depends on the composition, boundary conditions and theoretical description of the plasma.

We may represent any wave disturbance, $\boldsymbol{A}(\mathbf{x},t)$, by its Fourier components (with amplitude, $\boldsymbol{A}(\mathbf{k},\omega)$), wave vector $\mathbf{k}$, and frequency, ω):

$$\mathbf{A}(\mathbf{x},t) = \mathbf{A}(\mathbf{k},\omega)\,\exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$$

Phase velocity (wave front propagation)

$$\mathbf{v}_{ph} = \omega\mathbf{k}/k^2$$

Group velocity (energy flow)

$$\mathbf{v}_{gr} = \partial\omega/\partial\mathbf{k}$$

Wave-particle interactions

Plasma waves in a warm plasma interact with particles through:

- *Cyclotron resonance:* $\omega - \mathbf{k}\cdot\mathbf{v} = \pm\,\omega_{gi,e}$
- *Landau resonance:* $\omega - \mathbf{k}\cdot\mathbf{v} = 0$
- *Nonlinear particle trapping in large-amplitude waves*
- *Quasilinear particle (pitch-angle) diffusion*
- *Particle acceleration in turbulent wave fields*

There is a large variety of ***wave-particle interactions***. They may occur in connection with linear plasma instabilities, leading to ***wave growth and damping***, or take place in coherent or turbulent wave fields, leading to particle acceleration and heating.