Concepts of plasma micro- and macroinstability

- **Linear instability**
- **Beam plasma dispersion relation**
- **Two stream instability**
- **Rayleigh-Taylor instability**
- **Kelvin-Helmholtz instability**
- **Firehose and Mirror instability**
- **Flux tube instabilities**

Instabilities in a plasma

Because of a multitude of free-energy sources in space plasmas, a very large number of instabilities can develop.

If spatial the involved scale is:

- comparable to macroscopic size (bulk scale of plasma,.....)

-> ***macroinstability*** (affects plasma globally)

- comparable to microscopic scale (gyroradius, inertial length,...)

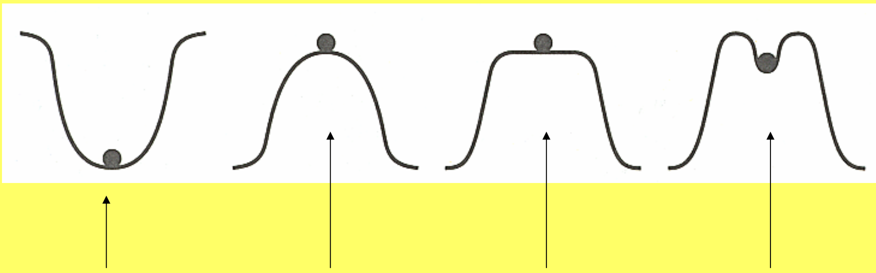
-> ***microinstability*** (affects plasma locally)

Theoretical treatment:

- *macroinstability*, fluid plasma theory
- *microinstability*, kinetic plasma theory

Concept of instability

Generation of instability is the general way of redistributing energy which was accumulated in a non-equilibrium state.



Linear instability

The concept of linear instability arises from the consideration of a linear wave function. Assume any variable (density, magnetic field, etc.) here denoted by A, the fluctuation of which is δA, that can be Fourier decomposed as

$$\delta A = \sum_{\mathbf{k}} A_{\mathbf{k}} \exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$$

In general the dispersion relation (DR) has complex solutions: $\omega = \omega_r + \gamma$. For real frequency the disturbances are oscillating waves. For complex solutions the sign of γ decides whether the amplitude A growths ($\gamma > 0$) or decays ($\gamma < 0$).

$$A_{\mathbf{k}}(t) = A_{\mathbf{k}} \exp\left[\gamma(\omega_r, \mathbf{k})t\right]$$

The linear approximation breaks down at the nonlinear time:

$$t_{n1} \approx \gamma^{-1} \ln\left(\frac{A_0}{A_{\mathbf{k}}}\right)$$

Weak instability

For the instability to remain ***linear*** we require the condition, $\gamma/\omega << 1$, to be fulfilled. In the opposite case ones speaks of a purley growing or non-oscillating instability. Generally, the waves obey a dispersion relation, $D(\omega, \mathbf{k}) = 0$, where

$$D(\omega,\mathbf{k}) = D_r(\omega,\mathbf{k}) + iD_i(\omega,\mathbf{k})$$

is a complex function. It is convenient to assume that the frequency is also complex: $\omega(\mathbf{k}) = \omega_r(\mathbf{k}) + i\gamma(\mathbf{k})$. For small growth rate the dispersion relation can be expanded in the complex plane about the real axis such that

$$\begin{aligned} D(\omega,\mathbf{k}) &= D_r(\omega_r,\mathbf{k}) + (\omega-\omega_r)\frac{\partial D_r(\omega,\mathbf{k})}{\partial\omega}|_{\gamma=0} \\ &+ iD_i(\omega_r,\mathbf{k}) = 0 \end{aligned}$$

The real frequency and growth rate are then obtained from:

$$D_r(\omega_r,\mathbf{k}) = 0$$

$$\gamma(\omega_r,\mathbf{k}) = -\frac{D_i(\omega_r\mathbf{k})}{\partial D_r(\omega,\mathbf{k})/\partial\omega|_{\gamma=0}}$$

Beam plasma dispersion relation

Consider the simplest electrostatic dispersion relation leading to instability, a cold plasma with background density n_0, and an electron beam with velocity, $\mathbf{v}_b$, and density, n_b. The wave frequencies are obtained (left as an exercise) by the zeros of the plasma response function, which reads:

$$\epsilon(\omega,\mathbf{k}) = 1 - \frac{\omega_{p0}^2}{\omega^2} - \frac{\omega_{pb}^2}{(\omega-\mathbf{k}\cdot\mathbf{v}_b)^2} = 0$$

Neglecting the drift yields simple Langmuir oscillations, and considering the beam only yields two ***beam modes***: $\omega(\mathbf{k}) = \mathbf{k}\,\mathbf{v}_b \pm \omega_{pb}$.

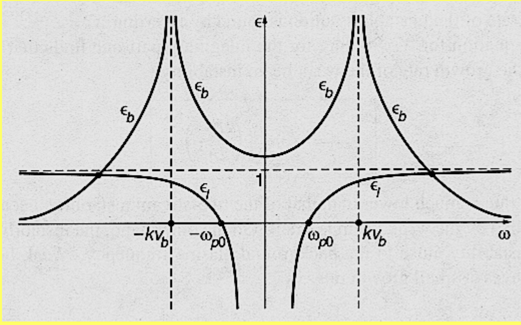
$$1 - \frac{\omega_{p0}^2}{\omega^2} = \frac{\omega_{pb}^2}{(\omega-\mathbf{k}\cdot\mathbf{v}_b)^2}$$

Two stream instability

The coupling between the (negative wave energy) beam mode and Langmuir waves leads to the ***two-stream instability***, having the dispersion:

$$1 - \frac{\omega_{p0}^2}{\omega^2} = \frac{\omega_{pb}^2}{(\omega - \mathbf{k}\cdot\mathbf{v}_b)^2}$$

which is graphically shown on the right side, and for which an approximate analytical solution (left as exercise) is obtained as:



$$\omega = \frac{kv_b}{2 + n_b/n_0}\left[1 \pm i\left(\frac{n_b}{n_0}\right)^{1/2}\right]$$

Buneman instability

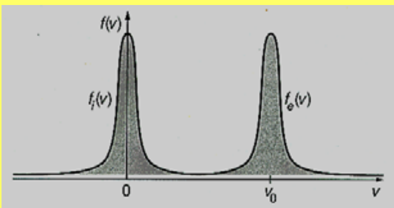
The electron-ion two-stream instability, ***Buneman instability***, arises from a DR that can be written as (with ions at rest and electrons at speed v_0):

$$\epsilon(\omega, \mathbf{k}) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(\omega - kv_0)^2} = 0$$

The velocity distribution is shown below (right). An approximate analytical solution (left as exercise) is obtained below (left). ***Sufficiently long wavelengths yield instability. Its growth rate is large, leading to violent current disruptions.***

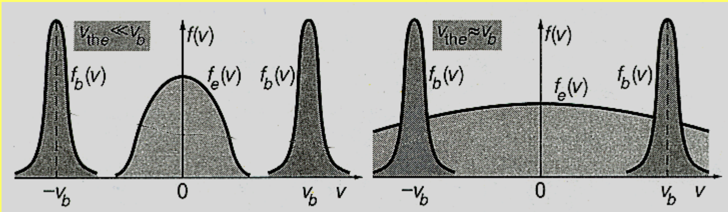
$$\omega_{bun} = \left(\frac{m_e}{16m_i}\right)^{1/3}\omega_{pe} \approx 0.03\omega_{pe}$$

$$\gamma_{bun} = \left(\frac{3m_e}{16m_i}\right)^{1/3}\omega_{pe} \approx 0.05\omega_{pe}$$



Counterstreaming ion beam instability

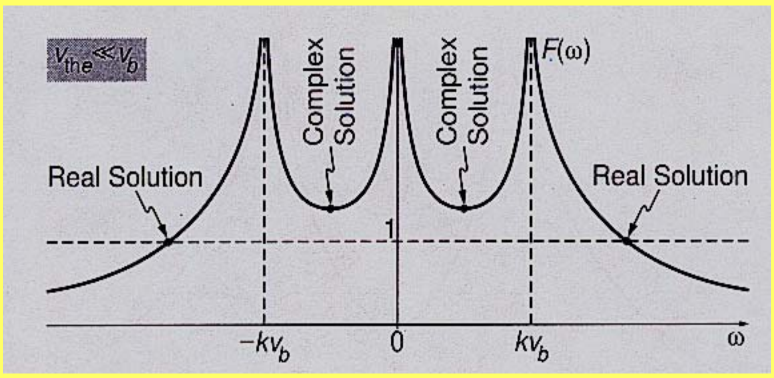
Counterstreaming ions (proton double beams) occur frequently in the solar wind and in front of the Earth bow shock. The configuration is illustrated below, with cold and hot background electrons.



The related dispersion relation, using the warm (v_{the} is the electron thermal speed) electron plasma dispersion function, *Z(ζ)*, is generally written as:

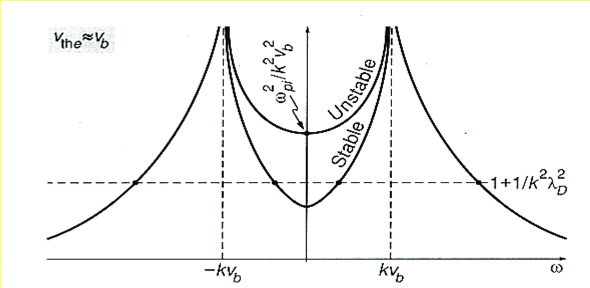
$$1 - \frac{1}{2k^2\lambda_D^2} Z'\left(\frac{\omega}{kv_{the}}\right) - \frac{1}{2}\left[\frac{\omega_{pi}^2}{(\omega - kv_b)^2} + \frac{\omega_{pi}^2}{(\omega + kv_b)^2}\right] = 0$$

Ion double-beam instability with cold electrons



$$1 - \frac{\omega_{pe}^2}{\omega^2} = \frac{1}{2}\left[\frac{\omega_{pi}^2}{(\omega - kv_b)^2} + \frac{\omega_{pi}^2}{(\omega + kv_b)^2}\right]$$

Ion beam instability with hot electrons



$$1+\frac{1}{k^2\lambda_D^2}=\frac{1}{2}\left[\frac{\omega_{pi}^2}{(\omega-kv_b)^2}+\frac{\omega_{pi}^2}{(\omega+kv_b)^2}\right]$$

The minium at $\omega = 0$ has the value $\omega_{pi}^2/(kv_b)^2$. Hence instability sets in if that minimum is above the horizontal line, a condition satisfied for small beam drifts only. ***Hot non-resonant electrons quench the instability.***

Rayleigh-Taylor instability I

It is the instability of a plasma boundary under the influence of a gravitational field. It is also called *gravitational instability*. If the attractive gravitation is replaced by the centrifugal force, the instability is called *flute instability*.

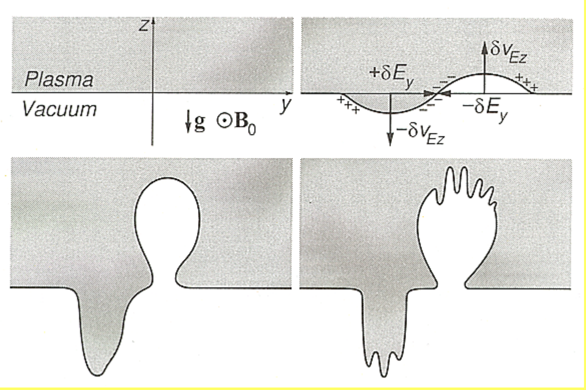
Consider a heavy plasma supported against gravity, $\mathbf{g} = -g\mathbf{e}_z$, by a magnetic field, $\mathbf{B}_0=B_0\mathbf{e}_x$, while the density gradient with scale L_n points upward, $\nabla n_0=\partial n_0(z)/\partial z\ \mathbf{e}_z$, and $\mathbf{g}\cdot\nabla n_0 < 0$.

The long-wavelength solution of the dispersion relation derived from a small perturbation of the ion fluid equations yields a purely growing mode with $\omega^2 = -g/L_n$. Growth time at the magnetopause is of the order of hours.

$$\gamma_{0rt}=\left(\frac{g}{L_n}\right)^{1/2}$$

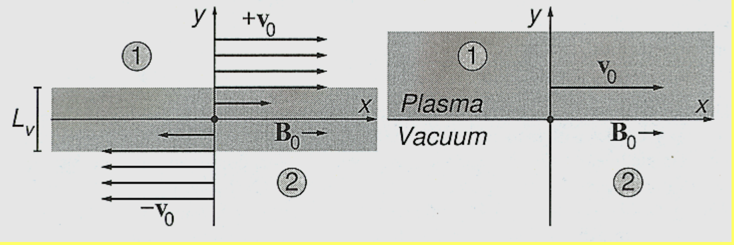
Rayleigh-Taylor instability II

- *Dilution of plasma caused by initial rarefaction rises up....*
- *Initial density enhancement below boundary falls.....*



Consider a distortion of the boundary so the plasma density makes a sinusoidal excursion. The gravitational field causes an ion drift and current in the negative y direction, $v_{iy} = -m_i g/(eB_0)$, in which electrons do not participate; -> charge separation electric field δE_y evolves. Opposing drifts amplify the original distortions. The bubbles develop similar distortions on even smaller scales.

Kelvin-Helmholtz instability I

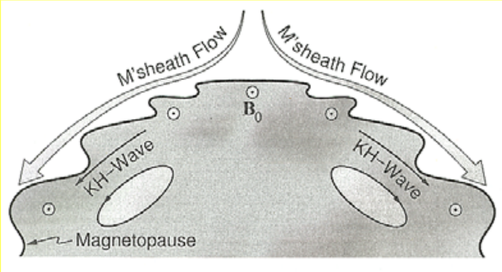


- ***Shear flow at magnetised plasma boundary may cause ripples on the surface that can grow.........***
- ***The rigidity of the field provides the dominant restoring force.......***

Consider shear flows (e.g., due to the solar wind) at a boundary, such as between Earth's magnetosheath and magnetopause. Linear perturbation analysis in both regions shows that ***incompressible waves confined to the interface*** can be excited, with the dispersion relation on the right:

$$\frac{1}{n_{02}\left[\omega^2-(\mathbf{k}\cdot\mathbf{v}_{A2})^2\right]} + \frac{1}{n_{01}\left[(\omega-\mathbf{k}\cdot\mathbf{v}_0)^2-(\mathbf{k}\cdot\mathbf{v}_{A1})^2\right]} = 0$$

Kelvin-Helmholtz instability II



Excitation of geomagnetic pulsations!

The dispersion relation is quadratic in ω and yields an unstable solution given by:

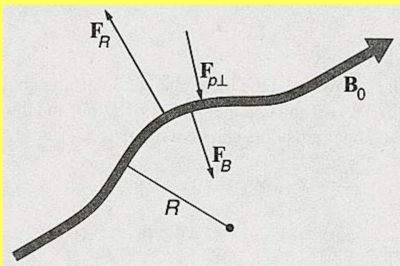
$$\omega_{kh} = \frac{n_{01}\mathbf{k}\cdot\mathbf{v}_0}{n_{01}+n_{02}}$$

corresponding to the appearance of a complex conjugate root if the streaming is large enough, i.e. if the subsequent inequality is fulfilled:

$$(\mathbf{k}\cdot\mathbf{v}_0)^2 > \frac{n_{01}+n_{02}}{n_{01}n_{02}}\left[n_{01}(\mathbf{k}\cdot\mathbf{v}_{A1})^2 + n_{02}(\mathbf{k}\cdot\mathbf{v}_{A2})^2\right]$$

Firehose instability I

Mechanism of the ***firehose instability***: Whenever the flux tube is slightly bent, the plasma exerts an outward centrifugal force (curvature radius, *R*), that tends to enhance the initial bending. The gradient force due to magnetic stresses and thermal pressure resists the centrifugal force. In force equilibrium we find:



$$\frac{m_i n_0 v_{th\|}^2}{R} = \frac{p_\perp}{R} + \frac{B_0^2}{\mu_0 R}$$

The resulting instability condition for breaking equilibrium is:

$$p_\| > p_\perp + B_0^2/\mu_0$$

Firehose instability II

The ***firehose instability*** may excite bulk long-wavelength Alfvèn waves in case of anisotropic plasma pressure:

$$\mathbf{P} = p_\perp \mathbf{I} + (p_\parallel - p_\perp)\frac{\mathbf{BB}}{B^2}$$

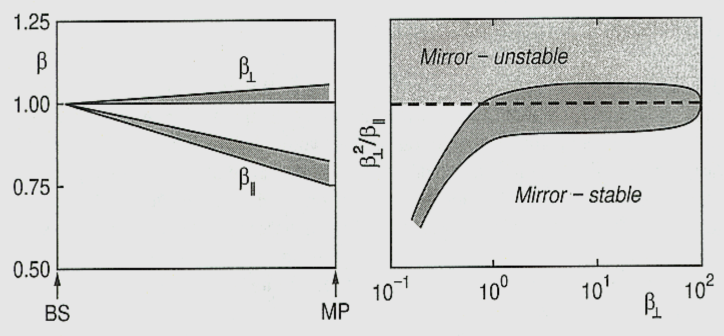
A magnetic flux tube may then be stimulated to perform transverse oscillations, like a firehose, at a growth rate obtained by perturbation of the anisotropic fluid equations.

$$\gamma_{fh} = \frac{k_\parallel v_A}{\sqrt{2}}\left(\beta_{0\parallel} - \beta_{0\perp} - 2\right)^{1/2}$$

The instability requires, $\beta_{||} > 2$, i.e. low fields like in the distant solar wind.

Mirror instability I

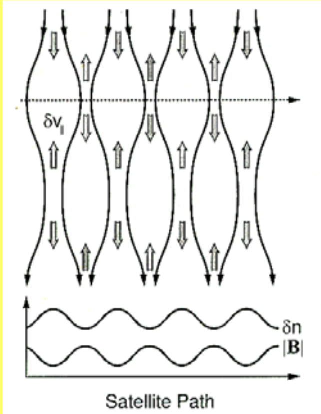
This long-wavelength ***compressive slow-mode instability*** requires consideration of particle motion parallel and perpendicular to the field and thus a kinetic treatment. Occurs in the Earth‘s dayside magnetosheet, where the shocked solar wind is heated adiabatically in the perpendicular direction, while the field-aligned outflow cools the plasma in the parallel direction (see Figure below with measured values in dark grey).



Mirror instability II

Sattellite measurements across a mirror-unstable region.

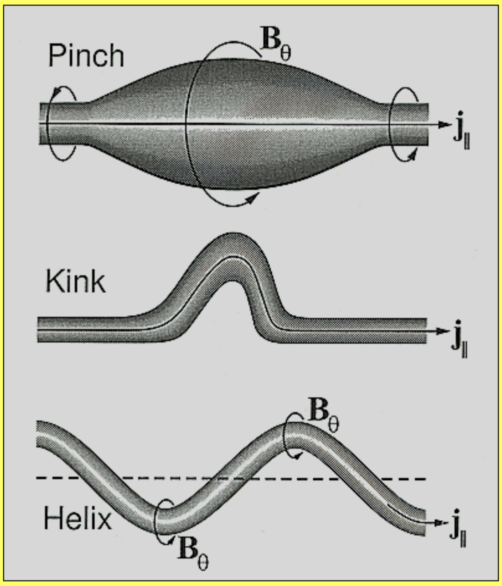
The particles stream into the mirror during instability, become trapped there and then oscillate between mirror points. Density and field out of phase, ***slow mode wave***!



The growth rate of the mirror mode results from kinetic theory:

$$\gamma_{mi} = \sqrt{\frac{2}{\pi}}\frac{\beta_{i\parallel}}{\beta_{i\perp}^2}\left[\sum_s \beta_{s\perp}\left(\frac{\beta_{s\perp}}{\beta_{s\parallel}} - 1\right) - 1\right] k_\parallel v_{thi\parallel}$$

Flux tube instabilities



Current disruption

Bending of magnetic field line

Spiral formation of thin flux tube