Kinetic plasma microinstabilities

- Gentle beam instability
- Ion- and electron-acoustic instability
- Current-driven cyclotron instability
- Loss-cone instabilities
- Anisotropy-driven waves
- Ion beam instabilities
- Cyclotron maser instability
- Drift-wave instability

Gentle beam instability I

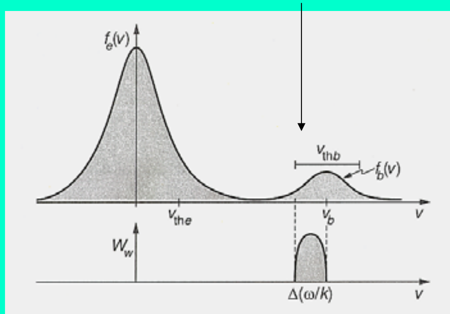
Electromagnetic waves can penetrate a plasma from outside, whereas electrostatic waves must be excited internally. The simplest kinetic instability is that of an electron beam propagating on a uniform background: ***gentle beam*** or ***bump-on-tail*** configuration:

$$\omega_l = \pm\omega_{pe}\left(1 + \frac{3}{2}k^2\lambda_D^2\right) + i\gamma_l(k)$$

Few fast electrons at speed $v_b >> v_{th0}$, but with $n_b << n_0$, can excite Langmuir waves.

$$\gamma_l = \omega_l \frac{\pi\omega_{pe}^2}{2n_0k^2}\frac{\partial f_{0e}(v)}{\partial v}\Big|_{v=\omega/k}$$

Positive gradient



Gentle beam instability II

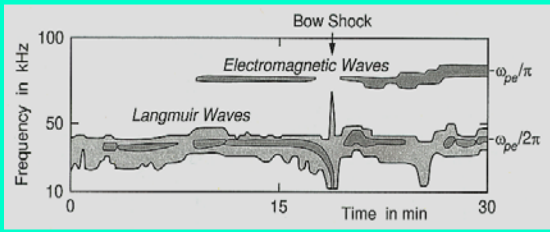
To calculate the growth rate (left as an exercise) of the ***gentle beam*** instability, we consider the sum of two Maxwellians:

$$f_{0e}(v) = f_0(v) + f_b(v - v_b)$$

The maximum growth rate is obtained for a cool, fast and dense beam.

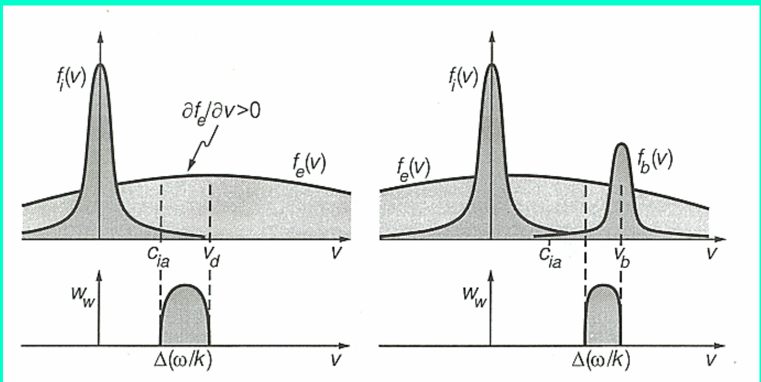
The condition for growth of Langmuir waves is that the beam velocity exceeds a threshold, $v_b > \sqrt{3}\, v_{th0}$, in order to overcome the Landau damping of the main part of the VDF. Electron beams occur in front of the ***bow shock*** and often in the solar corona during solar ***flares***.

$$\gamma_{gb,max} = \left(\frac{\pi}{2e}\right)^{1/2} \frac{n_b}{n_0} \left(\frac{v_b}{v_{thb}}\right)^2 \omega_l$$



Ion-acoustic instability I

Ion acoustic waves (electrostatic and associated with charge density fluctuations) can in principle be excited by ***electron currents*** or ***ion beams*** flowing across a plasma. Two unstable model VDFs are shown below.



The combined equilibrium distribution must have a positive slope at ω/k.

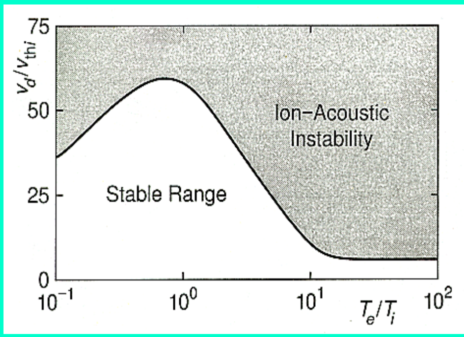
Ion-acoustic instability II

To calculate the growth rate of the ***current-driven*** electrostatic ion acoustic instability, we consider again the sum of two drifting Maxwellians. Using the ion-acoustic dispersion relation yields the growth rate:

$$\gamma_{ia} = \left(\frac{\pi}{8}\right)^{1/2} \frac{\omega_{ia}}{(1+k^2\lambda_D^2)^{3/2}} \left[\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{kv_d}{\omega_{ia}} - 1\right) - \left(\frac{T_e}{T_i}\right)^{3/2} \exp\left(-\frac{T_e}{T_i(1+k^2\lambda_D^2)}\right)\right]$$

Instability requirements at long-wavelengths:

- small ion Landau damping, $T_e >> T_i$
- large enough electron drift, $v_d >> c_{ia}$

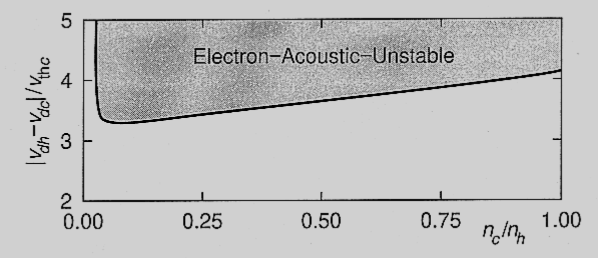


Electron-acoustic instability

Electron acoustic waves (electrostatic and associated with charge density fluctuations) can for example be generated by a two-component (hot and cold) drifting electron VDF, fulfilling the zero current condition: $n_h\mathbf{v}_{dh} + n_c\mathbf{v}_{dc} = \mathbf{0}$. The wave oscillates at the cold electron plasma frequency, $\omega_{ea} \approx \omega_{pc}$. Growth rate:

$$\gamma_{ea} \approx \left(\frac{\pi}{8}\right)^{1/2} \frac{\omega_{pc}}{k^2\lambda_{Dh}^2}\left(\frac{\mathbf{k}\cdot\mathbf{v}_{dh}}{kv_{thh}} - \frac{\omega_{ea}}{kv_{thh}}\right)\exp\left[-\frac{(\mathbf{k}\cdot\mathbf{v}_{dh} - \omega_{ea})^2}{k^2v_{thh}^2}\right]$$

Parameter space of the electron-acoustic instability



Current-driven cyclotron instabilities

Oblique ion-cyclotron waves

Parallel electron current along magnetic field, $\omega \approx l\omega_{gi}$,

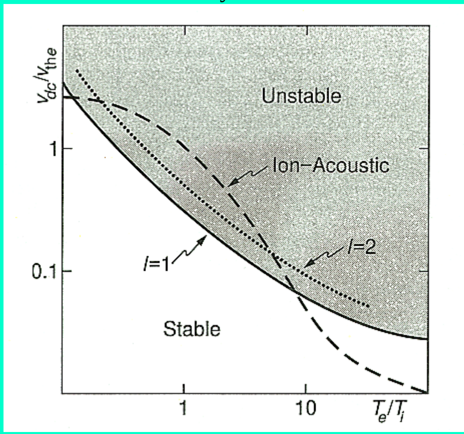
$k_{\|} << k_{\perp}$, $T_i << T_e$

Lower-hybrid modes

Transverse drift current across magnetic field, $\omega \approx \omega_{lh}$,

$\lambda << r_{gi}$ ions unmagnetized

Critical current drift speed for the ion-cyclotron wave

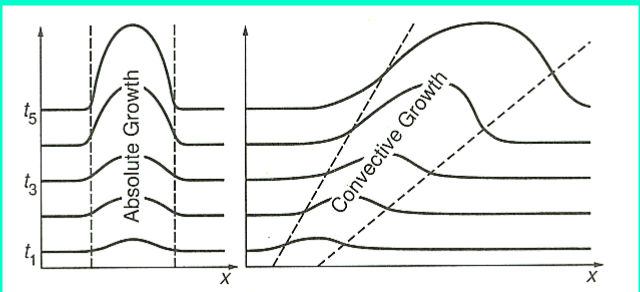


Currents can drive electrostatic ion-cyclotron and lower-hybrid modes.

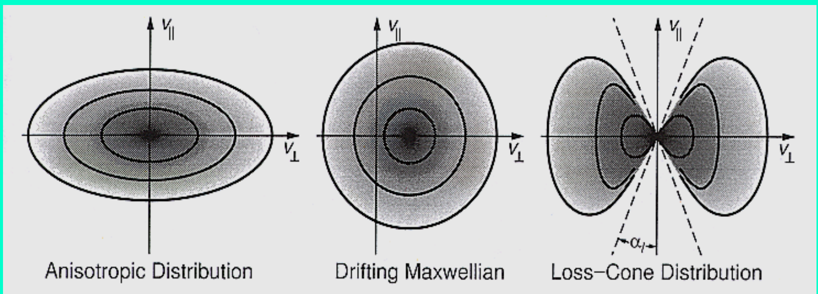
Absolute and convective instabilities

• ***Absolute or non-convective instability*:**
Wave energy stays at the locale of generation and accumulates; amplifies there with time of growth.

• ***Convective instability*:**
Wave energy is transported out of excitation site and disperses; amplifies only over that distance where growth rate is positive.



Velocity distributions containing free energy



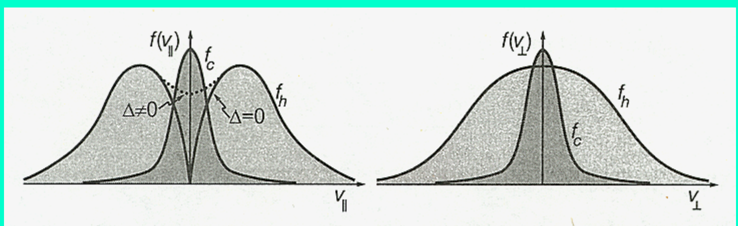
The most common anisotropic VDF in a uniform thermal plasma is the ***bi-Maxwellian*** distribution. Left figure shows a sketch of it with $T_\perp > T_{||}$.

$$f(v_\perp, v_\parallel) = \frac{n}{T_\perp T_\parallel^{1/2}} \left(\frac{m}{2\pi k_B}\right)^{3/2} \exp\left(-\frac{m v_\perp^2}{2k_B T_\perp} - \frac{m v_\parallel^2}{2k_B T_\parallel}\right)$$

Loss-cone instabilities

Electrostatic electron- and ion-cyclotron waves are very important electrostatic waves, because they occur at principle plasma resonances,

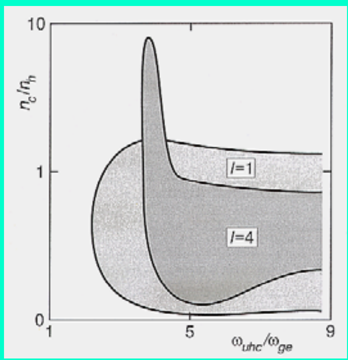
- contribute substantially to wave-particle energy exchange
- are purely of kinetic origin
- require for instability a particular shape of the VDF, with enhanced perpendicular energy, such as thermal anisotropy or a loss-cone

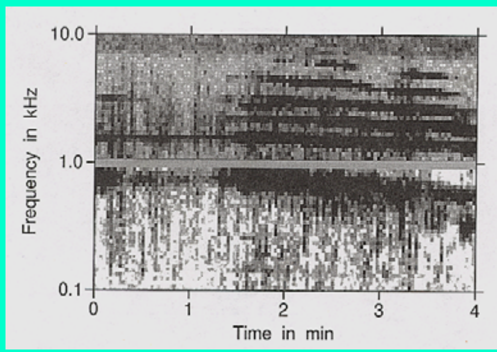


Loss-cone distributions store excess free energy in the gyromotion of the particles and are therefore well suited for exciting waves related to the cyclotron motion.

Electron-cyclotron loss-cone instability

Assume a cool neutralizing ion background (immobile), cold Maxwellian electrons and a ***hot dilute loss-cone component*** superposed. The dielectric response function is rather complicated (not suggested for an exercise). The region in parameter space of ***absolute instability*** is illustrated below (left). ***Multiple emitted harmonics*** of ω_{ge} as observed in the night-time equatorial magnetosphere are shown on the right.

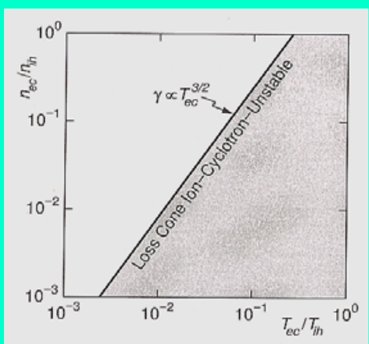


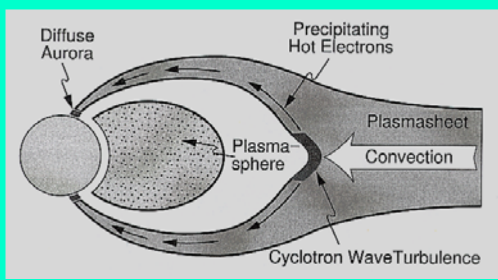


Electron-cyclotron harmonics are excited by a hot loss-cone distribution.

Ion-cyclotron loss-cone instability

Assume a cold neutralizing electron background and for the ions a cold component with a ***hot dilute loss-cone*** superposed. The region of instability in parameter space is illustrated below (left). Apparently, the instability depends also on the electron density and temperature. Ring current ions and electrons can, due to ***cyclotron wave turbulence,*** scatter into the loss cone and thus precipitate into the polar ionosphere and create aurora (right figure).

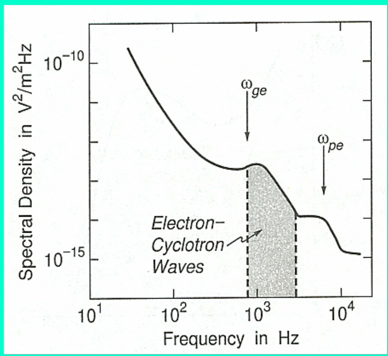




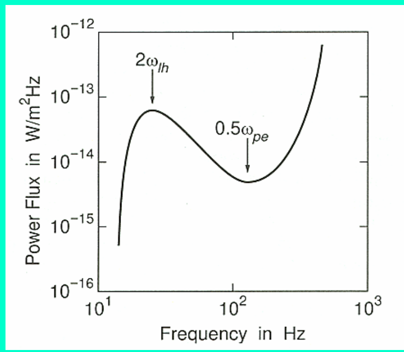
Ion-cyclotron harmonics are excited by a hot loss-cone distribution.

Plasma wave electric field spectra

Plasma sheet electron-cyclotron measured wave spectrum.
Excitation by loss-cone



Auroral hiss, broadband Whistler mode noise; emitted power calculated.
Excitation by field-aligned beam



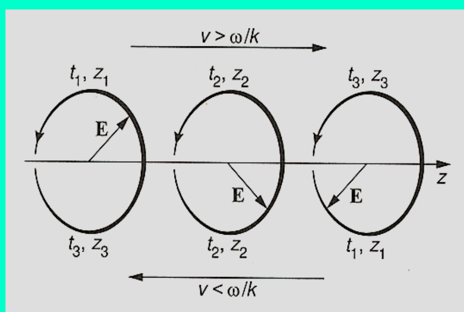
***Note that the typical electric field strength is only about* 10 μV/m *and the typical emitted power only a few* pW/m².**

Cyclotron resonance mechanism

Particles of a particular species with the right parallel velocity will see the wave ***electric field*** in their frame of reference with the ***suitable polarisation*** and thus undergo strong interaction with the wave. This is the nature of ***cyclotron resonance***, implying that

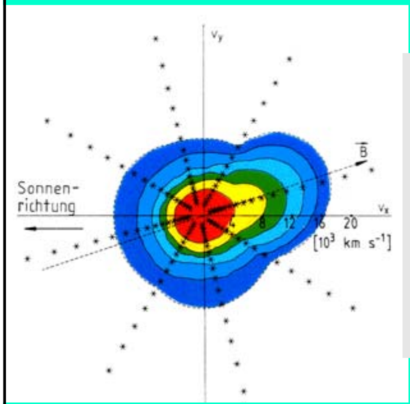
$$k_{\|} v_{\|} = \omega - l\omega_{gs}$$

The Doppler-shifted wave frequency (as e.g. seen by an electron) equals the lth harmonic of ω_{ge}. ***Being for l = 1 in perfect resonance, an electron at rest in the wave frame sees the wave at a constant phase.*** Otherwise, a slower (faster) electron will see the wave passing to the right (left), and thus it sees an (L) R-wave (not) polarised in the sense of its gyration.

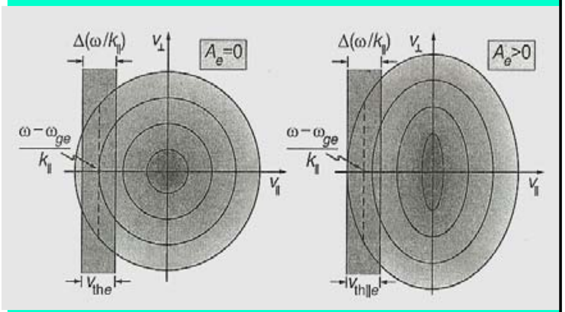


Whistler instability I

The ***resonance region*** for electrons in the Whistler instability is located in the negative $v_{\|}$ plane. An isotropic (left) and anisotropic model VDF with $A_e > 0$ ($A_e = T_{e\perp}/T_{e\|}$ -1) is shown. The width of the resonant region is about $v_{the\|}$.



Solar wind electrons



Model electron distributions

Whistler instability II

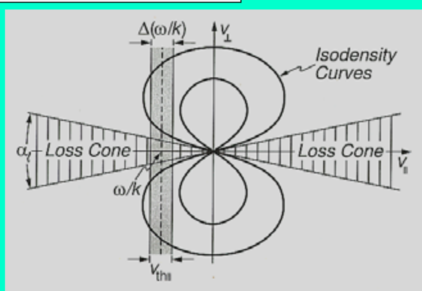
The resonance region for electrons in the ***Whistler instability*** is located in the negative $v_{\|}$ plane, opposite to the wave propagation direction. Consider a dense cold and a dilute hot and anisotropic (A_e) electron component, with $n_h << n_c$. Then the growth rate scales like, $\gamma \propto n_h/n_c$. The imaginary part of the dispersion reads:

$$D_i(\omega, k_{||}) = \frac{\sqrt{\pi}\omega}{|k_{||}|v_{thh||}} \frac{\omega_{ph}^2}{\omega^2} \left[1 - A_e\left(1 - \frac{\omega_{ge}}{\omega}\right)\right]$$

$$\times \exp\left[-\frac{(\omega-\omega_{ge})^2}{k_{||}^2 v_{thh||}^2}\right]$$

Instability requires that

$$\omega < \omega_c = \omega_{ge} A_e/(A_e + 1)$$

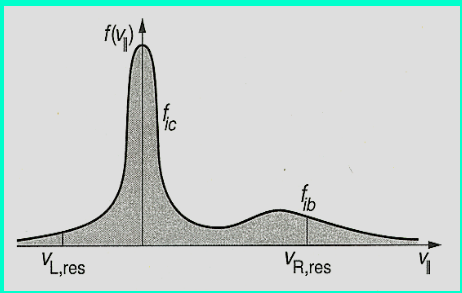


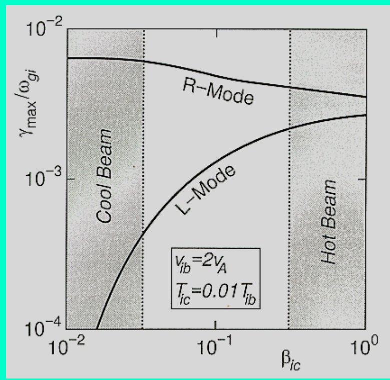
Resonance region and loss-cone VDF

Resonant ion beam instability

Consider an ***ion beam*** propagating along **B** as an energy source for low-frequency electromagnetic waves (see figure below, with a dense core and dilute beam, such that $n_b << n_c$) . The ***resonance speed*** for the ions is located in the negative $v_{||}$-plane for L-waves and positive $v_{||}$-plane for R-waves and given by:

$$v_{R,L,res} = (\omega \pm \omega_{gi})/k_{||}$$

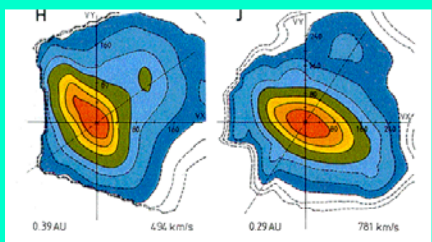




Maximum growth rate for dense core ions and a dilute ion beam

Solar wind proton beam and temperature anisotropy

The most prominent waves below the proton cyclotron frequency, $\omega << \omega_{gp}$ are electromagnetic ion cyclotron waves. They can be driven unstable e.g. by ***temperature anisotropies***, a free energy source which is most important and frequent in the solar wind (see the right figure).



For parallel propagation the dispersion relation for L and R waves (upper sign for RHP, and lower for LHP) reads:

$$N^2_{||R,L} = 1 - \frac{\omega^2_{pe}}{\omega(\omega \mp \omega_{ge})} - \frac{\omega^2_{pi}}{\omega(\omega \pm \omega_{gi})}$$

For $k_\perp = 0$, the electric field is perpendicular to* B*. Ions gyrate in the sense of L-modes, and electrons clockwise in the sense of R-modes.

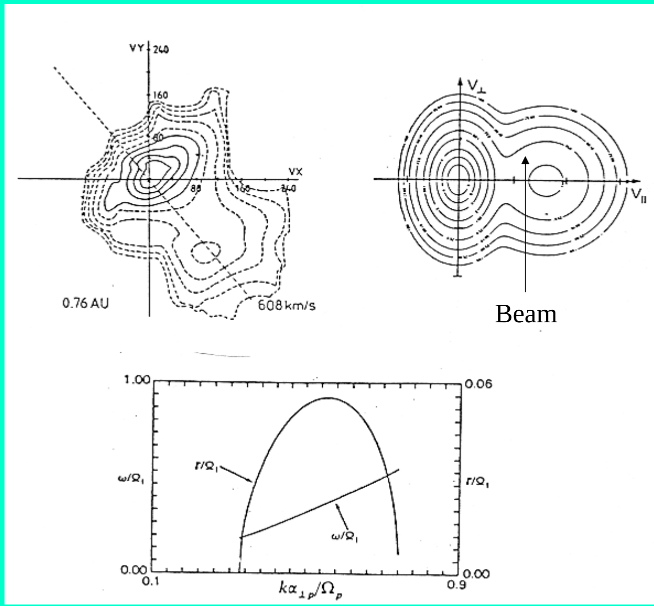
R-wave regulation of solar wind proton beam

- Measured (left) and modeled (right) velocity distribution
- Growth of fast mode R-waves
- ***Beam-driven instability***, large beam drift speed, v_b

$\omega \approx 0.4\omega_{gp}$

$\gamma \approx 0.06\omega_{gp}$

Marsch, 1991



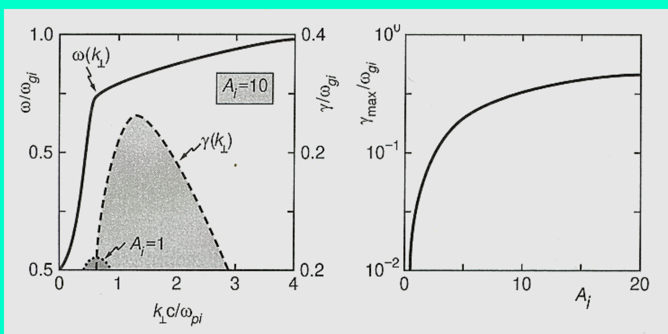
Ion cyclotron instability

The resonance region for ions in the ***cyclotron instability*** is located in the negative $v_{||}$ plane. A low-frequency instability can occur for $A_i > 0$ ($A_i = T_{i\perp}/T_{i||} - 1$), with the critical frequency given as:

$$\omega < \omega_c = \omega_{gi} A_i / (A_i + 1)$$

At parallel wavelengths shorter than the ion inertial scale, $k_{||}^2 >> (\omega_{pi}/c)^2$, the ***growth rate*** (shown below) can be comparatively large:

$$\gamma_{aic} \approx \omega_{gi} \sqrt{\beta_{i\perp}/2}$$



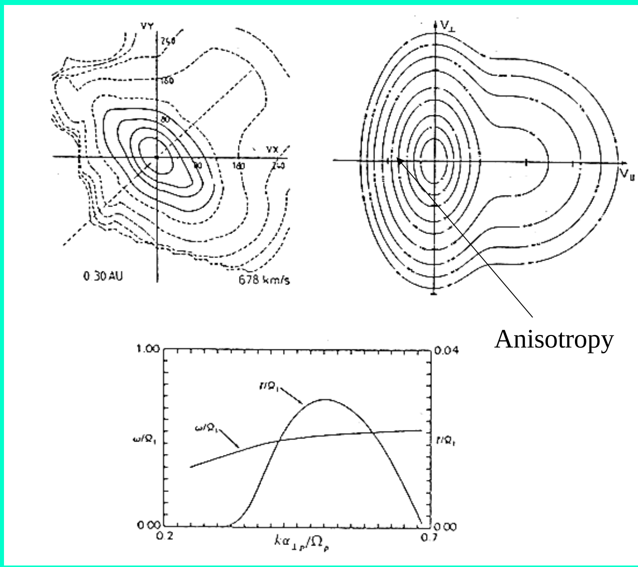
L-wave regulation of solar wind proton anisotropy

- Measured (left) and modeled proton (right) velocity distribution
- Growth of ion-cyclotron L-waves
- Anisotropy-driven instability by large perpendicular $T_\perp$

$\omega \approx 0.5\omega_{gp}$

$\gamma \approx 0.02\omega_{gp}$

Marsch, 1991



Cyclotron maser instability

Gyro- or synchrotron-emission of energetic (>10 keV) or ***relativistic electrons*** in ***planetary radiation belts*** can, while being trapped in the form of a loss-cone, lead to ***coherent free electromagnetic waves*** that can escape their source regions. Direct cyclotron emission fulfils the resonance condition:

$$k_{\|}v_{\|} - \omega + l\omega_{ge} = 0$$

In the relativistic case the dependence of ω_{ge} on the electron speed must be accounted for: ω_{ge} -> ω_{ge}/γ_R with the gamma factor:

$$\omega_{ge} \longrightarrow \omega_{ge}/\gamma_R$$

Expansion yields the quadratic equation for the resonance speed, which is an equation of a ***shifted circle***,

$$k_{\|}v_{\|} - \omega + l\omega_{ge}\left[1 - \left(v_{\|}^2 + v_{\perp}^2\right)/2c^2\right] = 0$$

along which the ***growth rate***, depending on $\partial f(v)/\partial v_\perp > 0$, has to be evaluated.