Collisions and transport phenomena

- **Collisions in partly and fully ionized plasmas**
- **Typical collision parameters**
- **Conductivity and transport coefficients**
- **Conductivity tensor**
- **Formation of the ionosphere and Chapman layer**
- **Heat conduction and viscosity**
- **Ionospheric currents**

Collisions

Plasmas may be *collisional* (e.g., fusion plasma) or *collisionsless* (e.g., solar wind). Space plasmas are usually collisionless.

Ionization state of a plasma:

• *Partially ionized*: Earth‘s ionosphere or Sun‘s photosphere and chromosphere, dusty and cometary plasmas

• *Fully ionized*: Sun‘s corona and solar wind or most of the planetary magnetospheres

Partly ionized, then ion-neutral collisions dominate; fully ionized, then Coulomb collisions between charge carriers (electrons and ions) dominate.

Collision frequency and free path

The neutral ***collision frequency***, ν_n, i.e. number of collisions per second, is proportional to the number of neutral particles in a column with a cross section of an atom or molecule, $n_n\sigma_n$, where n_n is the density and $\sigma_n = \pi d_0^2$ ($\approx 10^{-20}$ m^2) the atomic cross section, and to the average speed, $<\upsilon>$ ($\approx$ 1 km/s), of the charged particle.

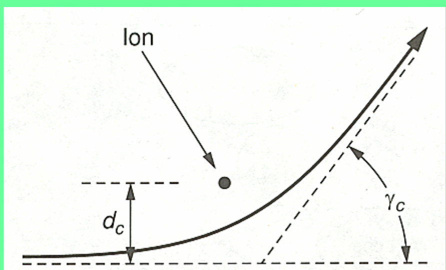
$$\nu_n = n_n \sigma_n \langle \upsilon \rangle$$

The ***mean free path length*** of a charged particle is given by:

$$\lambda_n = \frac{\langle \upsilon \rangle}{\nu_n} = (n_n \sigma_n)^{-1}$$

Coulomb collisions I

Charged particles interact via the Coulomb force over distances much larger than atomic radii, which enhances the cross section as compared to hard sphere collisions, but leads to a preference of small-angle deflections. Yet the potential is screened, and thus the interaction is cut off at the Debeye length, λ_D. ***The problem lies in determining the cross section, σ_c.***



$$\nu_{ei} = n_e \sigma_c \langle \upsilon_e \rangle$$

Impact or collision parameter, d_c, and scattering angle, γ_c.

Coulomb collisions II

The attractive Coulomb force exerted by an ion on an electron of speed v_e being at the distance d_c is given by:

$$F_C = -\frac{e^2}{4\pi\epsilon_0 d_c^2}$$

This force is felt by the electron during the fly-by time $t_c \approx d_c/\upsilon_e$ and thus leads to a momentum change of the size, $t_c F_C$, which yields:

$$|\Delta(m_e\upsilon_e)| \approx \frac{e^2}{4\pi\epsilon_0 \upsilon_e d_c}$$

For large deflection angle, $\gamma_c \approx 90^o$, the momentum change is of the order of the original momentum. Inserting this value above leads to an estimate of d_c, which is:

$$d_c \approx \frac{e^2}{4\pi\epsilon_0 m_e v_e^2}$$

Coulomb collisions III

The maximum cross section, $\sigma_c = \pi d_c^2$, can then be calculated and one obtaines the electron-ion collision frequency as:

$$\nu_{ei} = n_e\sigma_c\langle v_e\rangle \approx \frac{n_e e^4}{16\pi\epsilon_0^2 m_e^2 \langle v_e\rangle^3}$$

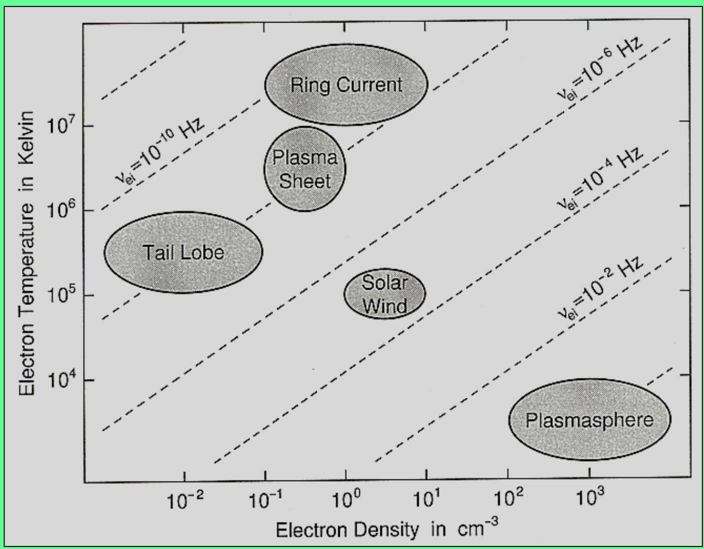
Taking the mean thermal speed for v_e, which is given by $k_B T_e = 1/2\, m_e v_e^2$, yields the expression:

$$\nu_{ei} \approx \frac{\sqrt{2}\omega_{pe}^4}{64\pi n_e}\left(\frac{k_B T_e}{m_e}\right)^{-3/2}$$

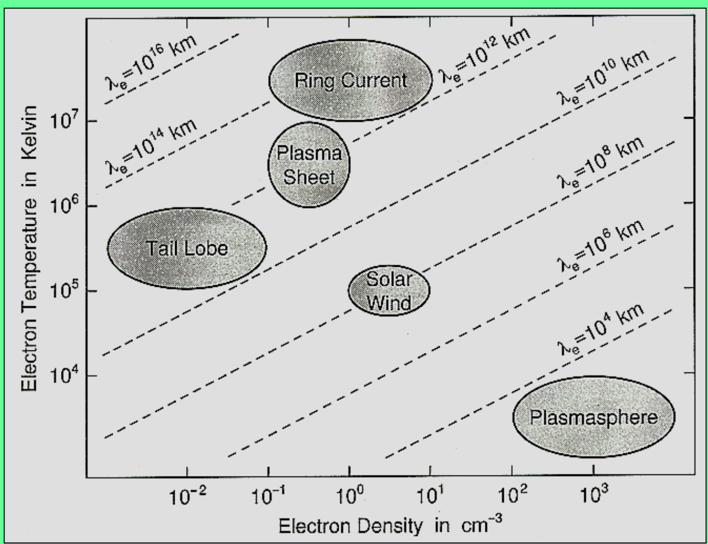
***The collision frequeny turns out to be proportional to the* (-3/2) *power of the temperature and to the density*.** A correction factor, $ln\Lambda$, still has to be applied to account for small angle deflections, where Λ is the plasma parameter, i.e. the number of particles in the Debye sphere.

$$\nu_{ei} \approx \frac{\omega_{pe}}{64\pi}\frac{ln\Lambda}{\Lambda}$$

Typical collision frequencies for geophysical plasmas



Coulomb mean free path lengths in space plasmas



Coulomb collisions in the solar wind

Parameter	Chromo -sphere	Corona ($1R_S$)	Solar wind (1AU)
N_e (cm^{-3})	10^{10}	10^7	10
T_e (K)	10^3	1-2 10^6	10^5
λ_e (km)	10	1000	10^7

***N* is the number of collisions between Sun and Earth orbit.**

- Since in fast wind $N < 1$, Coulomb collisions require kinetic treatment!
- Yet, only a few collisions ($N \geq 1$) remove extreme anisotropies!
- Slow wind: $N > 5$ about 10%, $N > 1$ about 30-40% of the time.

Plasma resistivity

In the presence of collisions we have to add a collision term in the equation of motion. Assume collision partners moving at velocity ***u***.

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - m\nu_c(\mathbf{v} - \mathbf{u})$$

In a steady state collisional friction balances electric acceleration. Assume there is no magnetic field, $\mathbf{B} = \mathbf{0}$. Then we get for the electrons with ions at rest:

$$\mathbf{E} = -\frac{m_e \nu_c}{e}\mathbf{v}_e$$

Since electrons move with respect to the ions, they carry the current density, $\mathbf{j} = -en_e v_e$. Combining this with the above equation yields, $\mathbf{E} = \eta\,\mathbf{j}$, with the resistivity:

$$\eta = \frac{m_e \nu_c}{n_e e^2}$$

Conductivity in a magnetized plasma I

In a steady state collisional friction balances the Lorentz force. Assume the ions are at rest, $\mathbf{v}_i = \mathbf{0}$. Then we get for the electron bulk velocity:

$$\mathbf{E} + \mathbf{v}_e \times \mathbf{B} = -\frac{m_e \nu_c}{e} \mathbf{v}_e$$

Assume for simplicity that, $\mathbf{B}=B\mathbf{e}_z$. Then we can solve for the electron bulk velocity and obtain the current density, which can in components be written as:

$$\begin{aligned} j_x &= \sigma_0 E_x + \frac{\omega_{ge}}{\nu_c} j_y \\ j_y &= \sigma_0 E_y - \frac{\omega_{ge}}{\nu_c} j_x \\ j_z &= \sigma_0 E_z \end{aligned}$$

$$\sigma_0 = \frac{n_e e^2}{m_e \nu_c}$$

Here we introduced the plasma conductivity (along the field).

The current can be expressed in the form of Ohm's law in vector notation as: $\mathbf{j} = \boldsymbol{\sigma} \mathbf{E}$, *with the dyadic conductivity tensor* $\boldsymbol{\sigma}$.

Conductivity in a magnetized plasma II

For a magnetic field in *z* direction the conductivity tensor $\boldsymbol{\sigma}$ reads:

$$\sigma = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{||} \end{pmatrix}$$

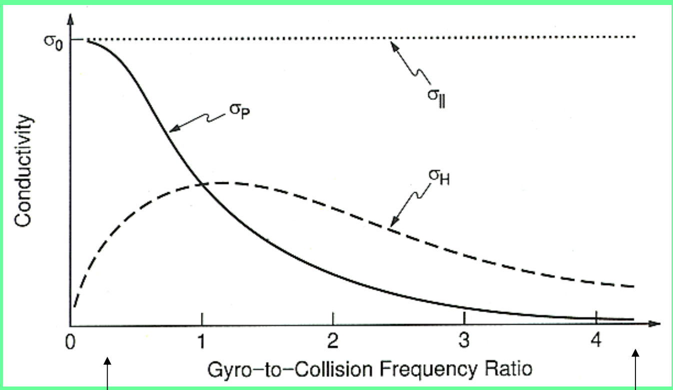
When the magnetic field has an arbitrary orientation, the current density can be expressed as:

$$\mathbf{j} = \sigma_{||}\mathbf{E}_{||} + \sigma_P \mathbf{E}_{\perp} - \sigma_H (\mathbf{E}_{\perp} \times \mathbf{B})/B$$

The tensor elements are the *Pedersen*, σ_D, the *Hall*, σ_H, and the *parallel* conductivity. In a weak magnetic field the Hall conductivity is small and the tensor diagonal, i.e. the current is then directed along the electric field.

$$\begin{aligned} \sigma_P &= \frac{\nu_c^2}{\nu_c^2 + \omega_{ge}^2} \sigma_0 \\ \sigma_H &= -\frac{\omega_{ge} \nu_c}{\nu_c^2 + \omega_{ge}^2} \sigma_0 \\ \sigma_{||} &= \sigma_0 = \frac{n_e e^2}{m_e \nu_c} \end{aligned}$$

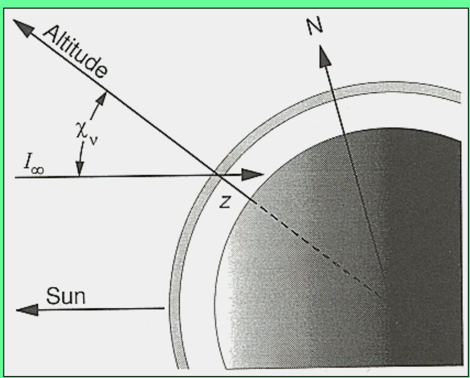
Dependence of conductivities on frequency ratio



$|\omega_{ge}| < \nu_c$, electrons are scattered in the field direction before completing gyration.

$|\omega_{ge}| > \nu_c$, electrons complete many gyrocircles before being scattered -> electric drift prevails.

Formation of the ionosphere



The ionosphere is the transition layer between the neutral atmosphere and ionized magnetosphere.

Solar ultraviolet radiation impinges at angle χ_ν, is absorbed in the upper atmosphere and creates ionization (also through electron precipitation). I_∞ is the flux on top of the layer.

The ionosphere is barometrically stratified according to the density law:

$$n_n(z) = n_0 \exp(-z/H)$$

H is the scale height, defined as, $H = k_B T_n / m_n g$, with g being the gravitational acceleration at height $z = 0$, where the density is n_0.

Diminuation of ultraviolet radiation

According to radiative transfer theory, the incident solar radiation is diminished with altitude along the ray path in the atmosphere:

$$dI = \sigma_\nu n_n \frac{dz}{\cos\chi_\nu} I$$

Here σ_ν is the radiation absorption cross section for radiation (photon) of frequency ν. Solving for the intensity yields:

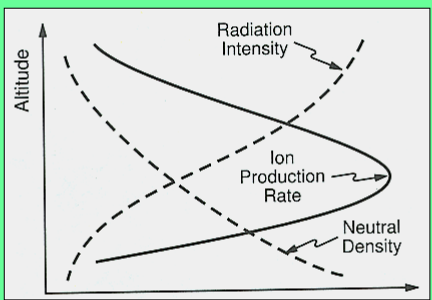
$$I(z) = I_\infty \exp\left[-\frac{\sigma_\nu n_0 H}{\cos\chi_\nu}\exp(-z/H)\right]$$

This shows the exponential decrease of the intensity with height, as is schematically plotted by the dashed line in the subsequent figure.

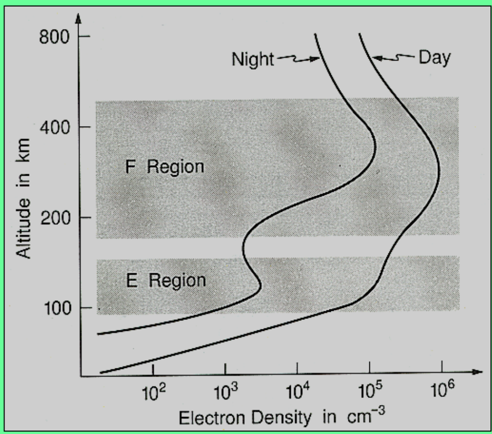
Formation of the Chapman layer

The number of electron-ion pairs locally produced by UV ionization, the photoionization rate per unit volume $q_\nu(z)$, is proportional to the ionization efficiency, κ_ν, and absorbed radiation: $q_\nu(z) = \kappa_\nu \sigma_\nu n_n I(z)$. This gives the Chapman production function, quoted and plotted below.

$$q_\nu(z) = \kappa_\nu \sigma_\nu n_0 I_\infty \exp\left[-\frac{z}{H} - \frac{\sigma_\nu n_0 H}{\cos\chi_\nu}\exp(-z/H)\right]$$



Electron recombination and attachment



Recombination, with coefficient α_r, and electron attachment, β_r, are the two major loss processes of electrons in the ionosphere.

In equilibrium quasi-neutrality applies:

$$n_e = n_i$$

Then the continuity equation for n_e reads:

$$\frac{dn_e}{dt} = q_{\nu,e} - \alpha_r n_e^2 - \beta_r n_e$$

Transport coefficients: Heat conduction and viscosity

Electrons in a collision-dominated plasma can carry heat in the direction of the temperature gradient,

Fourier's law: $\mathbf{Q}_e = - \kappa_e \nabla T_e$

$$\kappa_e = 5 n_e k_B^2 T_e / (2 m_e \nu_c)$$

Ions in a collision-dominated plasma can carry momentum in the direction of velocity gradients (shear, vorticity, etc..),

Viscous stresses: $\mathbf{\Pi}_i = - \zeta_i \, (\nabla \mathbf{V}_i + (\nabla \mathbf{V}_i)^T)$

$$\zeta_i = n_i k_B T_i / \nu_c$$

Ionospheric currents

Ions and electrons (to a lesser extent) in the E-region of the Earth ionosphere are coupled to the neutral gas. Atmospheric winds and tidal oscillations force the ions by friction to move across the field lines, while electrons move differently, which generates a current -> „dynamo“ layer driven by winds at velocity $\boldsymbol{v}_n$.
Ohm‘s law is modified accordingly:

$$\mathbf{j} = \sigma \cdot (\mathbf{E} + \mathbf{v}_n \times \mathbf{B})$$

Current systems:

- ***Current system created by atmospheric tidal motions***
- ***Equatorial electrojet (enhanced effective conductivity)***