Space Plasma Physics

Thomas Wiegelmann, 2012

1. Basic Plasma Physics concepts
2. Overview about solar system plasmas

Plasma Models

3. Single particle motion, Test particle model
4. Statistic description of plasma, BBGKY-Hierarchy and kinetic equations
5. Fluid models, Magneto-Hydro-Dynamics
6. Magneto-Hydro-Statics
7. Stationary MHD and Sequences of Equilibria

Statistical description of a plasma

- The complete statistic description of a system with N particles is given by the distribution function

$$F(x_1, x_2, \ldots, x_N, v_1, v_2, \ldots, v_N, t)$$

$$\int F \, dx_1, dx_2, \ldots, dx_N, dv_1, dv_2, \ldots, dv_N = 1$$

- Hyperspace for N particles has dimension 6 N +1
- N is typically very large (For Sun: N~ 10^{57})

=> No chance to compute or estimate F

Liouville Equation

after Joseph Liouville 1809-1889

The many body distribution

$$F(x_1, x_2, \ldots, x_N, v_1, v_2, \ldots, v_N, t)$$

obeys the Liouville equation

$$\frac{\partial F}{\partial t} + \sum_i \left(\frac{\partial F}{\partial x_i} \cdot v_i + \frac{\partial F}{\partial v_i} \cdot a_i^T \right) = 0$$

a_i^T acceleration of particle i due to external and interparticle forces

We define the one-particle distribution function $f_\alpha^{(1)}(x_1, v_1, t)$ by integrating $F(x_1, x_2, \ldots, x_N, v_1, v_2, \ldots, v_N, t)$ over coordinates and velocities of all but one particle of type α (say ions and electrons) and multiplying over number of particle N_α for each species

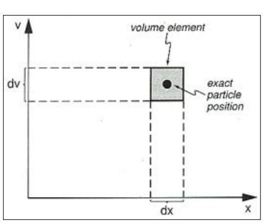
$$\overline{n}_\alpha f_\alpha^{(1)}(x_1, v_1, t) = N_\alpha \int F \, dx_2, \ldots, dx_N, dv_2, \ldots, dv_N$$

where $\overline{n}_\alpha = N_\alpha / V$ and V is the volume.

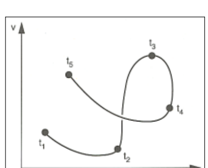
$\overline{n}_\alpha f_\alpha^{(1)} \, dx_1 \, dy_1$ is the number of particles at x_1 with velocity v_1 in the range $dx_1 \, dy_1$.

For plasmas in equilibrium $f_\alpha^{(1)}$ has a maxwellian distribution in velocity space. Space plasmas are, however, often far away from equilibrium and the distribution is non-maxwellian.

Phase space



Six-dimensional *phase space* with coordinates axes **x** and **v** and volume element d**x**d**v**

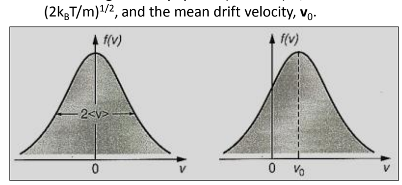


Many particles (i=1, N) having time-dependent position $\mathbf{x}_i(t)$ and velocity $\mathbf{v}_i(t)$. The particle path at subsequent times (t_1,.., t_5) is a curve in phase space

Maxwellian velocity distribution function

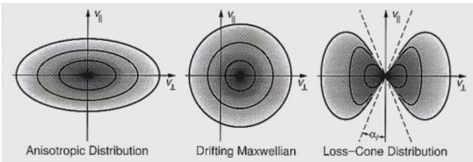
The general equilibrium VDF in a uniform thermal plasma is the *Maxwellian* (Gaussian) distribution.

The average velocity spread (variance) is, <v> = $(2k_BT/m)^{1/2}$, and the mean drift velocity, $\mathbf{v}_0$.



$$f(\mathbf{v}) = n \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp\left(-\frac{m(\mathbf{v} - \mathbf{v}_0)^2}{2k_B T} \right)$$

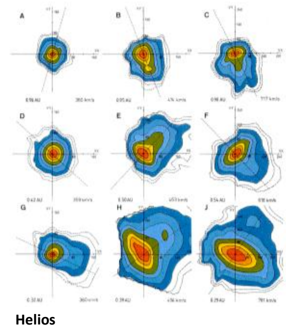
Anisotropic model velocity distributions



The most common anisotropic VDF in a uniform thermal plasma is the *bi-Maxwellian* distribution. Left figure shows a sketch of it, with $T_\perp > T_{||}$.

$$f(v_\perp, v_\|) = \frac{n}{T_\perp T_\|^{1/2}}\left(\frac{m}{2\pi k_B}\right)^{3/2} \exp\left(-\frac{mv_\perp^2}{2k_B T_\perp} - \frac{mv_\|^2}{2k_B T_\|}\right)$$

Measured solar wind proton velocity distributions

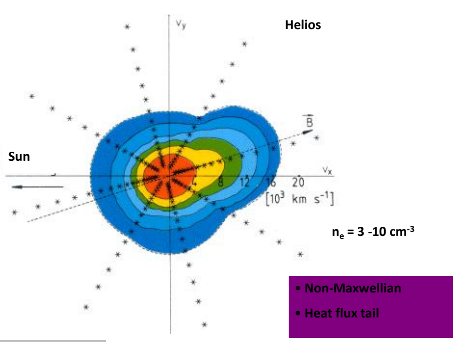


- Temperature anisotropies
- Ion beams
- Plasma instabilities
- Interplanetary heating

Plasma measurements made at 10 s resolution (> 0.29 AU from the Sun)

Marsch et al., JGR, **87**, 52, 1982

Measured solar wind electrons



Pilipp et al., JGR, **92**, 1075, 1987

Set of reduced distribution functions

We can define an entire set of reduced distribution functions from F, for example the two-particle distribution function $f^{(2)}_{\alpha,\gamma}(x_1, v_1, x_2, v_2, t)$ by integrating $F(x_1, x_2, \ldots, x_N, v_1, v_2, \ldots, v_N, t)$ over coordinates and velocities of all but two particles:

$$\overline{n}_\alpha \overline{n}_\gamma\, f^{(2)}_{\alpha,\gamma}(x_1, v_1, x_2, v_2, t) = N_\alpha N_\gamma \int F\, dx_3, \ldots, dx_N, dv_3, \ldots, dv_N$$

where α and γ can be the same type or different species. (say describing the interaction of 2 electrons, 2 ions, or 1 ion and 1 electron)

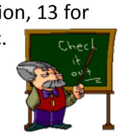
Similar we can define the three-particle distribution function and so on.

How to proceed?

$$\frac{\partial F}{\partial t} + \sum_i \left(\frac{\partial F}{\partial x_i}\cdot v_i + \frac{\partial F}{\partial v_i}\cdot a_i^T\right) = 0$$

- We integrate over the Liouville equation and get evolutionary equations for the reduced distribution functions.
- This reduces the number of dimensions from 6 N+1 to 7 for the one particle distribtion function, 13 for the two-particle distribution function etc.

=> Derivation on the blackboard



Set of reduced equations

After integrating the Lioville equation we get the evolutionary equation for the one particle distribution function.

closed form

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + v_1\cdot\frac{\partial f_\alpha^{(1)}}{\partial x_1} + a_1^E\frac{\partial f_\alpha^{(1)}}{\partial v_1} = \left.\frac{\partial f_\alpha^{(1)}}{\partial t}\right|_{\text{particle}}$$

where the term on the right side is due to particle interaction. We calculated (see blackboard) this term as :

$$\left.\frac{\partial f_\alpha^{(1)}}{\partial t}\right|_{\text{particle}} = -\sum_\beta \overline{n}_\beta \int a_{1,\beta}\frac{\partial}{\partial v_1} f^{(2)}_{\alpha,\beta}(x_1, v_1, x_\beta, v_\beta, t)\, dx_\beta\, dv_\beta$$

We do not know the two particle distribution function! How can we derive it?

- We reduced the high-dimensional (6N+1) Liouville equation to a set of equations for reduced distribution-functions.

Set of reduced equations

- Problem: Equation for the one particle distribution function contains the two particle distribution function on the right side.
- In principle we know, how to derive that one: Do the corresponding integration over the Liouville equation (we do not show that explicitely in this lecture)
- Problem: Equation for the two particle distribution function contains the 3-particle distribution function on right side, and so on.

BBGKY-Hierarchy
(Bogoliubov,Born,Green,Kirkwood,Yvon)

The full set of these equations is equivalent to the Liouville-equation.

- Problem: Equation for $f^{(n)}$ contains $f^{(n+1)}$.
- $\Rightarrow$ We cannot solve the full set of equations in the BBGKY-hierarchy. This is as complicated as solving the Liouville-equation directly.
- How to proceed?

Remark: This hierarchy of equations was first published in french by J. Yvon (1935), but this work was hardly recognized that time and got only attention after it was re-discovered in the end of the 1940th.

BBGKY-Hierarchy

- We must cut-off the hierarchy at some point.
- This means we make a suitable assumption for the term containing $f^{(n+1)}$, without computing it exactly.
- For a plasma we cut already after the first equation (for $f^{(1)}$) and make assumptions regarding $f^{(2)}$

$\Rightarrow$ **Kinetic Equations**.

- We have to simplify the term

$$\sum_\beta \bar{n}_\beta \int a_{1,\beta} \frac{\partial}{\partial v_1} f^{(2)}_{\alpha,\beta}(x_1, v_1, x_\beta, v_\beta, t)\, dx_\beta\, dv_\beta$$

Kinetic Equations

- Particle interactions in a plasma are long range and we divide the particle interaction forces in:
  - Average force due to many distant particles.
  - Force due to nearest neighbours (Collisions).
- The average forces due to many distant particles do not depend on the exact position of these individual particles and we treat them together with the external forces.

$$\mathbf{a} = \mathbf{a}_{\text{ext}} + \langle \mathbf{a}_{\text{int}} \rangle$$

Kinetic Equations

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + \mathbf{v_1} \cdot \frac{\partial f_\alpha^{(1)}}{\partial \mathbf{x_1}} + \frac{q_\alpha}{m_\alpha} \langle \mathbf{E} + \mathbf{v_1} \times \mathbf{B} \rangle \cdot \frac{\partial f_\alpha^{(1)}}{\partial \mathbf{v_1}} = \left. \frac{\partial f_\alpha^{(1)}}{\partial t} \right|_c$$

~~where we get for the binary-collision rate~~

$$\left. \frac{\partial f_\alpha^{(1)}}{\partial t} \right|_c = -\sum_\beta \int \left(\mathbf{a}_{1,\beta} - \langle \mathbf{a}^{\text{int}}_{1,\beta} \rangle \right) \cdot \frac{\partial}{\partial \mathbf{v_1}} f^{(2)}_{\alpha,\beta}(x_1, v_1, x_\beta, v_\beta, t)\, dx_\beta\, dv_\beta$$

Simplest approach: Neglect the binary-collision rate. Only the average forces created by the other particles are considered => Collisionless plasma => Vlasov equation

Two problems remain:

- How do we get the average fields $\langle \mathbf{E} \rangle$ and $\langle \mathbf{B} \rangle$?
- The collision term $\left. \frac{\partial f_\alpha^{(1)}}{\partial t} \right|_c$ still contains $f^{(2)}$.

Vlasov Equation

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + \mathbf{v_1} \cdot \frac{\partial f_\alpha^{(1)}}{\partial \mathbf{x_1}} + \frac{q_\alpha}{m_\alpha} \langle \mathbf{E} + \mathbf{v_1} \times \mathbf{B} \rangle \cdot \frac{\partial f_\alpha^{(1)}}{\partial \mathbf{v_1}} = 0$$

We get average fields $\langle \mathbf{E} \rangle$ and $\langle \mathbf{B} \rangle$ from Maxwell equations

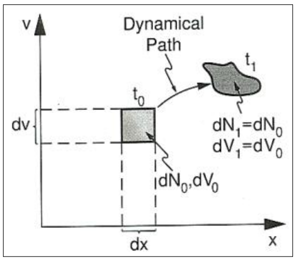
$$\nabla \cdot \langle \mathbf{E} \rangle = \frac{\langle \rho_q \rangle}{\epsilon_0}$$

$$\nabla \times \langle \mathbf{B} \rangle = \mu_0 \langle \mathbf{J} \rangle + \mu_0 \epsilon_0 \frac{\partial \langle \mathbf{E} \rangle}{\partial t} = 0$$

How do we get the average charge density $\langle \rho_q \rangle$ and the average electric current $\langle \mathbf{J} \rangle$?

Anatoly Vlasov
(1908-1975)

Vlasov equation



The Vlasov equation expresses phase space density conservation. A 6D-volume element evolves like in an incompressible fluid.

Vlasov equation is nonlinear via closure with Maxwell's equations.

Vlasov Equation

- We have actually to solve two Vlasov equations: for ions $f_i^{(1)}$ and electrons $f_e^{(1)}$.
- We write short f_i and f_e from now on and also $\mathbf{x}$ and $\mathbf{v}$ instead of $\mathbf{x_1}$ and $\mathbf{v_1}$.
- The fields $\langle \mathbf{E} \rangle$ and $\langle \mathbf{B} \rangle$ are of course unique and couple these two Vlasov equations.
- The average charge density $\langle \rho_q \rangle$ and the average electric current $\langle \mathbf{J} \rangle$ are only functions of the location and time, but not the velocity space.
- $\langle \rho_q \rangle = \langle \rho_{qi} \rangle + \langle \rho_{qe} \rangle$, $\langle \mathbf{J} \rangle = \langle \mathbf{J_i} \rangle + \langle \mathbf{J_e} \rangle$
- We have to relate the macroscopic quantities $\langle \rho_q(\mathbf{x}, t) \rangle$ and $\langle \mathbf{J}(\mathbf{x}, \mathbf{t}) \rangle$ to the distribution functions $f_i(\mathbf{x}, \mathbf{v}, t)$ and $f_e(\mathbf{x}, \mathbf{v}, t)$.

Macroscopic variables of a plasma

- We take moments of the distribution function f_α, where α stands for ions and electrons. Moment means that we integrate in velocity space over quantities like

$$\int \mathbf{v}^n f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

- Zero moment, densities (particle-, mass-, charge density):

$$n_\alpha(\mathbf{x}, t) = \overline{n}_\alpha \int f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

$$\rho_{m\,\alpha}(\mathbf{x}, t) = \overline{n}_\alpha\, m_\alpha \int f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

needed in Vlavov-Maxwell system

$$\rho_{q\,\alpha}(\mathbf{x}, t) = \overline{n}_\alpha\, q_\alpha \int f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

Macroscopic variables of a plasma

- First moment (particle flux $\mathbf{\Gamma}$, macroscopic plasma flow $\mathbf{V}$, electric current density $\mathbf{J}$):

$$\Gamma_\alpha(\mathbf{x}, t) = \overline{n}_\alpha \int \mathbf{v}\, f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} = n_\alpha(\mathbf{x}, t)\, \mathbf{V}_\alpha$$

$$\mathbf{V}_\alpha(\mathbf{x}, t) = \frac{\int \mathbf{v}\, f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}}{\int f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}}$$

$$J_\alpha(\mathbf{x}, t) = q_\alpha\, \overline{n}_\alpha \int \mathbf{v}\, f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} = q_\alpha\, n_\alpha(\mathbf{x}, t)\, \mathbf{V}_\alpha(\mathbf{x}, t)$$

needed in Vlavov-Maxwell system

Macroscopic variables of a plasma

- Second moment (Pressure Tensor P, scalar pressure p):

$$P_\alpha(\mathbf{x}, t) = \overline{n}_\alpha\, m_\alpha \int (\mathbf{v} - \mathbf{V}_\alpha)\, (\mathbf{v} - \mathbf{V}_\alpha)\;\; f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

- For spherical symmetric velocity distributions the pressure tensor becomes diagonal

$$P_\alpha(\mathbf{x}, t) = \begin{pmatrix} p_\alpha & 0 & 0 \\ 0 & p_\alpha & 0 \\ 0 & 0 & p_\alpha \end{pmatrix}$$

with the scalar pressure

$$p_\alpha(\mathbf{x}, t) = \frac{\overline{n}_\alpha\, m_\alpha}{3} \int (\mathbf{v} - \mathbf{V}_\alpha)^2\, f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} = n_\alpha\, k_b\, T_\alpha$$

The assumption of a scalar pressure is popular for it's simplicity, but not valid in some space plasmas like the solar wind. This leads to an anisotropic pressure tensor.

Macroscopic variables of a plasma

- Third moment (Heat flux $\mathbf{H}$):

$$\mathbf{H}_\alpha(\mathbf{x}, t) = \frac{\overline{n}_\alpha\, m_\alpha}{2} \int \mathbf{v}\, (\mathbf{v} \cdot \mathbf{v})\, f_\alpha(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$$

- It is possible to compute additional moments, but we cannot necessarily relate these higher moments to physical quantities.
- First and second moments are sufficient to close the Vlasov-Maxwell system.
- We can derive also equations for the derived macroscopic quantities. This leads to a fluid description of the plasma (like MHD) and we do that soon.
- In the following we continue to study the Vlasov-Maxwell system, kinetic equations and make more sophisticated approaches for collisions.

Vlasov-Maxwell Equations
kinetic description of a collisionless plasma

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}} = 0$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\sum_\alpha \overline{n}_\alpha q_\alpha \int f_\alpha \, d\mathbf{v} + \frac{\rho_{q\,\mathrm{ext}}}{\epsilon_0}$$

$$\nabla\times\mathbf{B} = \mu_0 \sum_\alpha \overline{n}_\alpha q_\alpha \int \mathbf{v}\, f_\alpha \, d\mathbf{v} + \mu_0\,\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}_{\mathrm{ext}}$$

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

α stands for ions and electrons.

Properties of Vlasov Equation

- Vlasov Equation conserves particles
- Distribution functions remains positive
- Vlasov equation has many equilibrium solutions

$$\mathbf{v}\cdot\frac{\partial f_{\alpha\,0}}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_{\alpha\,0}}{\partial \mathbf{v}} = 0$$

Index 0 stands for equilibrium. Any distribution function $f_{\alpha\,0}$ which depends only on constants of motions (say $a(x,v)$, $b(x,v)$, ...) of the particle trajectories solves the stationary Vlasov equation. (show proof on blackboard)

$$f_{\alpha\,0}(x,v) = f_{\alpha\,0}\,(a(x,v), b(x,v), \dots)$$

Stability of Vlasov equilibria

- $\frac{\partial f_{\alpha 0}}{\partial t} = 0$ does not guaranty stability of $f_{\alpha\,0}$.
- Assume a nearby disturbed case

$$f_\alpha(t) = f_{\alpha\,0} + \delta f_\alpha(t)$$

- How does it develop with the time-dependent Vlasov-equation?
  - If $\delta f_\alpha(t)$ decreases in time, the system will finally reach $f_{\alpha\,0} \Rightarrow$ Stable
  - If $\delta f_\alpha(t)$ increases, the system will get further away from $f_{\alpha\,0} \Rightarrow$ Unstable
  - If f decreases monotonically with v^2 then the equilibrium is stable (Proof on blackboard):

$$\frac{\partial f}{\partial v^2} < 0$$

Example: The well known Maxwell-Boltzmann-distribution

$$f(\mathbf{v}) \propto \exp\left(-\frac{m\,v^2}{2\,k_b\,T}\right)$$

is stable!

(Proof on blackboard)

James Clerk Maxwell, 1831-1871

Ludwig Boltzmann, 1844-1906

Further Examples

- Drifting Maxwellian ?

$$f(\mathbf{v}) \propto \exp\left(-\frac{m\,(\mathbf{v}-\mathbf{u_D})^2}{2\,k_b\,T}\right)$$

- Maxwellian with a non-thermal feature ?

$$f(v) =\propto \exp\left(-\frac{m\,v^2}{2\,k_b\,T}\right) + \epsilon\exp\left(-10\cdot\frac{m\,(\mathbf{v}-\mathbf{u_D})^2}{2\,k_b\,T}\right)$$

Investigate stability as an excercise.

Kinetic equations of first order

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + \mathbf{v_1}\cdot\frac{\partial f_\alpha^{(1)}}{\partial \mathbf{x_1}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v_1}\times\mathbf{B})\cdot\frac{\partial f_\alpha^{(1)}}{\partial \mathbf{v_1}} = \left.\frac{\partial f_\alpha^{(1)}}{\partial t}\right|_c$$

where we get for the binary-collision rate

$$\left.\frac{\partial f_\alpha^{(1)}}{\partial t}\right|_c = -\sum_\beta \int \left(\mathbf{a}_{1,\beta} - \langle \mathbf{a}_{1,\beta}^{\mathrm{int}}\rangle\right)\cdot\frac{\partial}{\partial \mathbf{v_1}} f_{\alpha,\beta}^{(2)}(x_1, v_1, x_\beta, v_\beta, t)\, dx_\beta\, dv_\beta$$

- Vlasov approach: neglect 2 particle distribution function completely
- Now we make a more sophisticated approach, the Mayer cluster expansion.

Mayer cluster expansion

$$f^{(2)}_{\alpha,\beta} = \underbrace{f^{(1)}_{\alpha}(x_1, v_1, t)\, f^{(1)}_{\beta}(x_2, v_2, t)}_{\text{Particles are independent from each other}} + \underbrace{g^{(2)}_{\alpha,\beta}(x_1, v_1, x_2, v_2, t)}_{\text{Correlation of the 2 particles}}$$

- In Vlasov theory we assumed $g^{(2)}_{\alpha,\beta} = 0$, which means that the single particle distribution functions are uncorrelated.
- We take now correlations into account, but assume

$$g^{(2)}_{\alpha,\beta} \ll f^{(1)}_{\alpha}\, f^{(1)}_{\beta}$$

This is a valid assumption, because in a small volume

$$V \ll \Lambda^3_D$$

The joint distribution function $f^{(2)}_{\alpha,\beta}$ is determined by the many particles outside V and not by the seperation of two particle from each other.

Collisions

Plasmas may be *collisional* (e.g., fusion plasma) or *collisionsless* (e.g., solar wind). Space plasmas are usually collisionless. => Vlasov equation

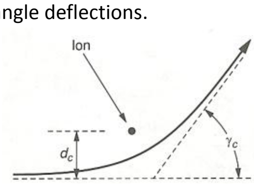
Ionization state of a plasma:

• *Partially ionized*: Earth's ionosphere or Sun's photosphere and chromosphere, dusty and cometary plasmas
=> Collisions with neutrals dominate

• *Fully ionized*: Sun's corona and solar wind and most of the planetary magnetospheres
=> Coulomb collisions or collisionless

Coulomb collisions

Charged particles interact via the Coulomb force over distances much larger than atomic radii, which enhances the cross section as compared to hard sphere collisions, but leads to a preference of small-angle deflections.



$$\nu_{ei} = n_e \sigma_c \langle v_e \rangle$$

Impact or collision parameter, d_c, and scattering angle, γ_c.

Models for the collision terms

Neutral-ion collisions are described by a Maxwell Boltzmann collision terms as in Gas-dynamics. (f_n : distribution function of neutrals, ν_n collision frequency):

$$\left(\frac{\partial f}{\partial t}\right)_c = \nu_n (f_n - f)$$

Coulomb Collisions lead to the Fokker-Planck-Equation which can often be described as a diffusion process.

$$\left(\frac{\partial f}{\partial t}\right)_c = \nabla_{\mathbf{v}} \cdot (\mathbf{D} \cdot \nabla_{\mathbf{v}} f)$$

Further sophistications are to take into account that Coulomb collisions occur not in vaccum, but are Debye-shielded and to consider wave-particle interactions.

Kinetic Equations

- In Kinetic theory we have a statistic description in a 6D phase space (configuration and velocity space).
- Kinetic equations are a first order approximation of the BBGKY-Hierarchy.
- Equation for Particle distribution function is coupled with Maxwell-equations => Difficult to solve.
- Most space plasmas are collisionless => Vlasov equation
- Colission terms depend on nature of process:
  - collision with neutrals => Maxwell-Boltzmann
  - Coloumb collisions: Fokker-Planck
  - Coloumb c. with Debye-shielding => Lennard Balescu

How to proceed?

- Due to the nonlinear coupling with Maxwell-equations, the Kinetic equations are difficult to solve.
- In numerical simulations one has to resolve relevant plasma scales like Debye-length, gyro-radii of electrons and ions and also the corresponding temporal scales (gyro-frequencies, plasma frequencies), which are orders of magnitude smaller and faster as the macroscopic scales (size of magnetosphere or active region)
- We often not interested in details of the velocity distribution function.
- => Integration over the velocity space lead to fluid equations like MHD (3D instead of 6D space)