

Space Plasma Physics

Thomas Wiegmann, 2012

1. Basic Plasma Physics concepts

2. Overview about solar system plasmas

Plasma Models

3. Single particle motion, Test particle model
4. Statistic description of plasma, BBGKY-Hierarchy and kinetic equations
5. Fluid models, Magneto-Hydro-Dynamics
6. Magneto-Hydro-Statics
7. Stationary MHD and Sequences of Equilibria

Used Material

- Lecture notes from Eckart Marsch 2007
 - Baumjohann&Treumann: Basic Space Plasma Physics
 - Schindler: Physics of space plasma activity
 - Priest: Solar MHD
 - Kulsrud: Plasma Physics for Astrophysics
 - Krall & Trivelpiece: Principles of Plasma Physics
 - Chen: Introduction to plasma physics and controlled fusion
 - Balescu: Plasma Transport (3 volumes)
 - Spatschek: Theoretische Plasmaphysik
- Wikipedia, Google and YouTube

Space Plasma Physics

Physical Processes

8. Plasma Waves, instabilities and shocks
9. Magnetic Reconnection

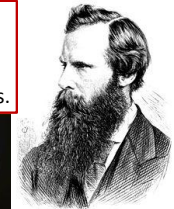
Applications

10. Planetary Magnetospheres
11. Solar activity
12. Transport Processes in Plasmas



What is plasma?

In plasma physics we study ionized gases under the influence of electro-magnetic fields.



Levi Tonks (1897-1971) and Irving Langmuir (1881-1957, photo) first used the term „**plasma**“ for a collection of charged particles (Phys. Rev. 1929)

William Crookes (1832-1919) called ionized matter in a gas discharge (Crookes-tube, photo) „**4th state of matter**“ (Phil. Trans 1879)

Source: Wikipedia



Neon-lights, Fluorescent lamps, Plasma globes

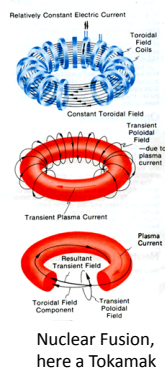
Industrial Plasmas



Electric Arcs



Semi conductor device fabrication



Ball lightning

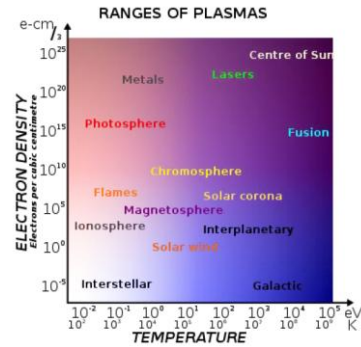
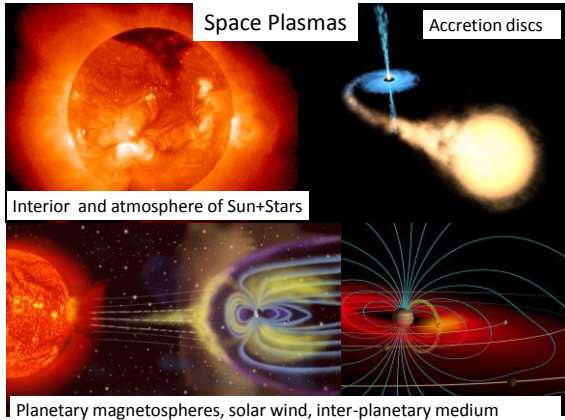
Natural Plasmas on Earth



Lightning

Aurorae

Source: Wikipedia



Source: Wikipedia

Typical ranges of plasma parameters: orders of magnitude (OOM)

Characteristic	Terrestrial plasmas	Cosmic plasmas
Size in meters	10 ⁻⁶ m (lab plasmas) to 10 ² m (lightning) (~8 OOM)	10 ⁻⁶ m (spacecraft sheath) to 10 ²⁵ m (intergalactic nebula) (~31 OOM)
Lifetime in seconds	10 ⁻¹² s (laser-produced plasma) to 10 ⁷ s (fluorescent lights) (~19 OOM)	10 ¹ s (solar flares) to 10 ¹⁷ s (intergalactic plasma) (~16 OOM)
Density in particles per cubic meter	10 ⁷ m ⁻³ to 10 ³² m ⁻³ (inertial confinement plasma)	1 m ⁻³ (intergalactic medium) to 10 ³⁰ m ⁻³ (stellar core)
Temperature in kelvins	~0 K (crystalline non-neutral plasma ^[10]) to 10 ⁸ K (magnetic fusion plasma)	10 ² K (aurora) to 10 ⁷ K (solar core)
Magnetic fields in teslas	10 ⁻⁴ T (lab plasma) to 10 ³ T (pulsed-power plasma)	10 ⁻¹² T (intergalactic medium) to 10 ¹¹ T (near neutron stars)

Source: Wikipedia

Source: Wikipedia

Comparison: Gas and Plasma

Property	Gas	Plasma
Electrical conductivity	Very low: Air is an excellent insulator until it breaks down into plasma at electric field strengths above 30 kilovolts per centimeter ^[14]	Usually very high: For many purposes, the conductivity of a plasma may be treated as infinite.
Independently acting species	One: All gas particles behave in a similar way, influenced by gravity and by collisions with one another.	Two or three: Electrons, ions, protons and neutrons can be distinguished by the sign and value of their charge so that they behave independently in many circumstances, with different bulk velocities and temperatures, allowing phenomena such as new types of waves and instabilities.
Velocity distribution	Maxwellian: Collisions usually lead to a Maxwellian velocity distribution of all gas particles, with very few relatively fast particles.	Often non-Maxwellian: Collisional interactions are often weak in hot plasmas and external forcing can drive the plasma far from local equilibrium and lead to a significant population of unusually fast particles.
Interactions	Binary: Two-particle collisions are the rule, three-body collisions extremely rare.	Collective: Waves, or organized motion of plasma, are very important because the particles can interact at long ranges through the electric and magnetic forces.

Plasmas studied in this lecture

- Non-relativistic particle velocities $v \ll c$
- Spatial and temporal scales are large compared to Planck length (1.6 10⁻³⁵ m) and time (5.4 10⁻⁴⁴ s)
More precise: Any action variables like (momentum x spatial dimension, Energy x time) are large compared to Planck constant ($h = 6.6 \cdot 10^{-34}$ Js)
Classic plasma, no quantum-mechanic effects.
- Plasmas violating these conditions (Quark-Gluon Plasma, relativistic plasma) are active areas of research, but outside the scope of this introductory course.

What is a plasma?

- A fully or partly ionized gas.
- Collective interaction of charged particles is more important than particle-particle collisions.
- Charged particles move under the influence of electro-magnetic fields (Lorentz-force)
- Charged particles cause electric fields, moving charged particles cause electric currents and thereby magnetic fields (Maxwell equations).

What is a plasma?

- In principle we can study a plasma by solving the Lorentz-force and Maxwell-equations selfconsistently.
 - With typical 10^{20} - 10^{50} particles in space plasmas this is not possible. (Using some 10^9 or more particles with this approach is possible on modern computers)
- => Plasma models

Plasma models

- **Test particles:**
Study motion of individual charged particles under the influence of external electro-magnetic (EM) fields
- **Kinetic models:**
Statistic description of location and velocity of particles and their interaction + EM-fields. (Vlasov-equation, Fokker-Planck eq.)
- **Fluid models:**
Study macroscopic quantities like density, pressure, flow-velocity etc. + EM-fields (MHD + multifluid models)
- **Hybrid Models:** Combine kinetic + fluid models

Maxwell equations for electro-magnetic fields

The motion of charged particles in space is strongly influenced by the self-generated electromagnetic fields, which evolve according to **Ampere's and Faraday's** (induction) laws (in this lecture we use the SI-system):

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

where ϵ_0 and μ_0 are the vacuum dielectric constant and free-space magnetic permeability, respectively. The charge density is ρ and the current density \mathbf{j} . The electric field obeys **Gauss** law and the magnetic field is always free of divergence, i.e. we have:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0$$

$$\nabla \cdot \mathbf{B} = 0$$

Electro-static effects

A plasma is quasi-neutral

- On large scales the positive (ions) and negative charges cancel each other.
- On small scales charge separations occur.
Can we estimate these scales?
=> Do calculations on blackboard.



Electromagnetic forces

The motion of charged particles in space is determined by the electrostatic **Coulomb** force and magnetic **Lorentz** force:

$$\mathbf{F}_C = q\mathbf{E}$$

$$\mathbf{F}_L = q(\mathbf{v} \times \mathbf{B})$$

where q is the charge and \mathbf{v} the velocity of any charged particle. If we deal with electrons and various ionic species (index, s), the **charge and current densities** are obtained, respectively, by summation over all kinds of species as follows:

$$\rho = \sum_s q_s n_s$$

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v}_s$$

which together obey the **continuity equation**, because the number of charges is conserved, i.e. we have:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

Debye-length

$$\Lambda_D = \sqrt{\frac{3\epsilon_0 k_B T}{2n^2}} \propto \sqrt{\frac{T}{n}}$$

Remark: Some text-books drop the factor $\sqrt{3/2} \sim 1.2$

Debye-length

- Electric potential for a charge in vacuum:

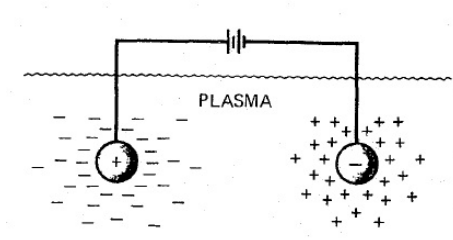
$$\Phi_0 = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

- Electric potential in a plasma:

$$\Phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \exp\left(-\frac{r}{\Lambda_D}\right)$$

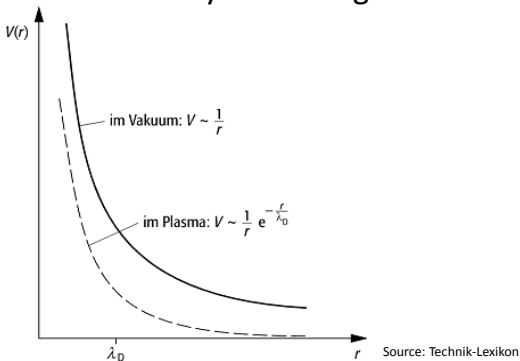
=> Debye shielding

Debye shielding



Source: Chen, Wikipedia

Debye shielding



Plasma Oscillations

- How does a charged particle (say an electron) move in a non-magnetized plasma?
- Solve electrostatic Maxwell equation selfconsistently with equation of motion, here for the Coulomb force
- => Do calculations on blackboard.



Plasma Oscillations

- Plasma frequency

$$\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}} \propto \sqrt{n}$$

- Electron plasma frequency is often used to specify the electron density of a plasma. How? => Dispersion relation of EM-waves See lecture of plasma waves.

$$\omega_{pe} = \sqrt{\frac{ne^2}{m_e\epsilon_0}} \propto \sqrt{n_e}$$

Plasma parameter

- The plasma parameter g indicates the number of particles in a Debye sphere.

$$g = \frac{1}{n \Lambda_D^3} \propto \frac{n^{1/2}}{T^{3/2}}$$

- Plasma approximation: $g \ll 1$.
- For effective Debye shielding and statistical significance the number of particles in a Debye sphere must be high.

Plasma parameter g

- g gives a measure how collective effects dominate over single particle effects.
- Contrast between neutral gas and plasma:
 - Interaction region of a neutral atom is the atomic radius R and $n R^3 \ll 1$
 - Interaction region in a plasma is the Debye sphere and $1/n \Lambda_D^3 \ll 1$
- Plasma state can be derived from expansion of exact many body equations with g.

Collisions: Mean free path

- The mean free path is the distance a particle moves in average before it suffers a collision.
- Cross section σ for interaction of particles during collisions in a plasma is approximated with the Debye-length.
- Mean free path: $l_{mfp} = \frac{1}{n \sigma}$
- Collision frequency: $\nu_c = n \sigma v$
 $\nu_c = n \langle \sigma(v) v \rangle$

Magnetized Plasmas

In order to study the transport of plasma and magnetic field lines quantitatively, let us combine **Maxwell's** equations with the simple phenomenological **Ohm's law**, relating the electric field in the plasma frame with its current:

$$\mathbf{j} = \sigma_0(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

(Later we will derive Ohm's law more systematically from kinetic plasma theory)

Induction equation

Using **Maxwell equations** for slow time variations, without the displacement current yields the induction equation (with constant conductivity σ_0):

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_{\text{Convection}} + \underbrace{\frac{1}{\mu_0 \sigma_0} \nabla^2 \mathbf{B}}_{\text{Diffusion}}$$



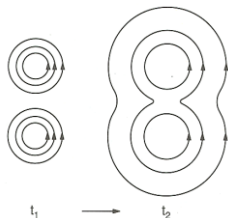
Exercise: Derive this equation for a spatial constant conductivity. How does the equation look if the conductivity varies spatially?

Magnetic diffusion

Assuming the plasma be at rest, the induction equation becomes a pure diffusion equation:

$$\frac{\partial \mathbf{B}}{\partial t} = D_m \nabla^2 \mathbf{B}$$

with the magnetic diffusion coefficient $D_m = (\mu_0 \sigma_0)^{-1}$.

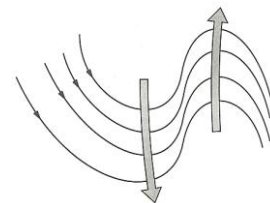


Under the influence of finite resistivity the magnetic field diffuses across the plasma, and field inhomogenities are smoothed out at time scale, $\tau_r = \mu_0 \sigma_0 L_B^2$, with scale length L_B .

Hyromagnetic theorem

In an ideal collisionless plasma in motion with infinite conductivity the induction equation becomes:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$



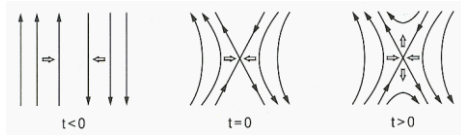
The field lines are constrained to move with the plasma -> **frozen-in field**. If plasma patches on different sections of a bundle of field lines move oppositely, then the lines will be deformed accordingly. Electric field in plasma frame, $\mathbf{E}' = \mathbf{0}$, -> voltage drop around closed loop is zero.

Magnetic reconnection

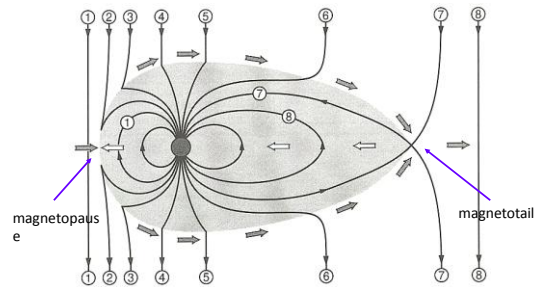
Assuming the plasma streams at bulk speed V , then the induction equation can be written in simple dimensional form as:

$$\frac{B}{\tau} = \frac{VB}{L_B} + \frac{B}{\tau_d}$$

The ratio of the first to second term gives the so-called **magnetic Reynolds number**, $R_m = \mu_0 \sigma_0 L_B V$, which is useful to decide whether a plasma is diffusion or convection dominated. Current sheet with converging flows -> magnetic merging at points where $R_m \approx 1$. Field lines form **X-point** and **separatrix**.



Field line merging and reconnection in the Earth's magnetosphere

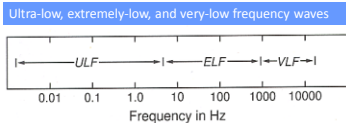


Waves in plasmas

In a plasma there are many reasons for spatio-temporal variations (waves or more generally **fluctuations**): High temperature required for **ionization** ($\Phi_H = 13.6 \text{ eV} \approx 158000 \text{ K}$) implies fast **thermal particle motion**. As a consequence

- > **microscopic fluctuating charge separations and currents**
- > **fluctuating electromagnetic fields.**

There are also **externally imposed disturbances** which may propagate through the plasma and spread their energy in the whole plasma volume. The relevant frequency ranges are:



Plasma waves are not generated at random. To exist they must satisfy two conditions:

- > **their amplitude must exceed the thermal noise level**
- > **they must obey appropriate dynamic plasma equations**

There is a large variety of **wave modes** which can be excited in a plasma. The mode structure depends on the composition, boundary conditions and theoretical description of the plasma.

We may represent any wave disturbance, $\mathbf{A}(\mathbf{x}, t)$, by its Fourier components (with amplitude, $\mathbf{A}(\mathbf{k}, \omega)$, wave vector \mathbf{k} , and frequency, ω):

$$\mathbf{A}(\mathbf{x}, t) = \mathbf{A}(\mathbf{k}, \omega) \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t)$$

Phase velocity (wave front propagation)

$$v_{ph} = \omega / k^2$$

Group velocity (energy flow)

$$v_{gr} = \partial\omega / \partial\mathbf{k}$$

Wave-particle interactions

Plasma waves in a warm plasma interact with particles through:

- **Cyclotron resonance:** $\omega - \mathbf{k} \cdot \mathbf{v} = \pm \omega_{g/e}$
- **Landau resonance:** $\omega - \mathbf{k} \cdot \mathbf{v} = 0$
- **Nonlinear particle trapping in large-amplitude waves**
- **Quasilinear particle (pitch-angle) diffusion**
- **Particle acceleration in turbulent wave fields**

There is a large variety of **wave-particle interactions**. They may occur in connection with linear plasma instabilities, leading to **wave growth and damping**, or take place in coherent or turbulent wave fields, leading to particle acceleration and heating.

Summary

- Plasma is a quasi-neutral ionized gas moving under the influence of EM-fields.
- Thermal energy of particles is much larger as potential energy (free particles).
- Quasi-neutrality can be violated in Debye-sphere.
- To qualify as plasma, spatial dimensions must be much larger as the Debye-sphere and many particles are in the sphere for effective shielding.
- Collision frequency must be much smaller as the plasma frequency.