Space Plasma Physics

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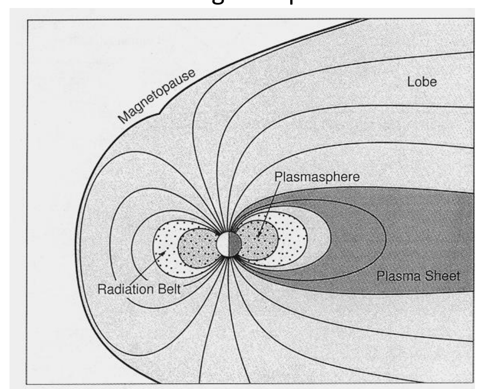
Physical Processes

8. Plasma Waves, instabilities and shocks
9. Magnetic Reconnection

Applications

10. Planetary Magnetospheres
11. Solar activity
12. Transport Processes in Plasmas

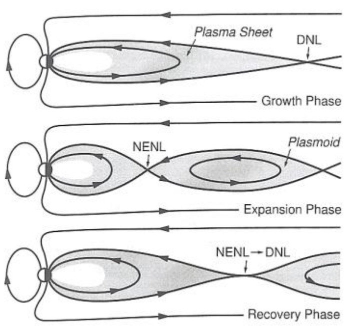
Earth magnetosphere

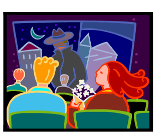


Magnetospheric substorm

Substorm phases:

- Growth
- Onset and expansion
- Recovery





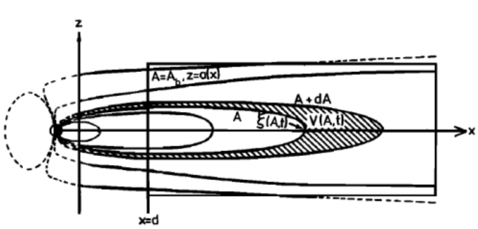
Show movie:
http://www.youtube.com/watch?v=BDZj1CmsJ64

Growth phase of magnetotail

- The growth phase (quiet evolution of magnetotail before a substorm) can be modelled as a sequence of magneto-static equilibria.
- Magnetic pressure in the lobe changes slowly (solar wind compresses field lines) and remains in equilibrium with plasma pressure in the plasma sheet.
- The magnetic field topology does not change in this phase (ideal MHD, no resistivity).
- Sequence of equilibria are not independent and the ideal MHD-evolution has to be implemented in the model.

Magnetospheric convection in quiet states

(Source: Schindler & Birn, JGR 1982)



Noon-midnight meridian plane of the magnetosphere.
V(A) is proportional to the differential volume between field lines A and A + dA.

Ideal MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$ Continuity Eq.

$$\rho \frac{\partial \underline{v}}{\partial t} + \rho \underline{v} \cdot \nabla \underline{v} = -\nabla p + \underline{j} \times \underline{B}$$ Force-balance

$$\underline{E} + \underline{v} \times \underline{B} = 0$$ Ideal Ohms Law

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^{\gamma}}\right) + \underline{v} \cdot \nabla\left(\frac{p}{\rho^{\gamma}}\right) = 0$$ Equation of state, Adiabatic convection

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{B} = \mu_0 \underline{j}$$

$$\nabla \cdot \underline{B} = 0$$

Maxwell

Source: Schindler & Birn, JGR 1982

Basic assumptions for magnetotail

- 2D configuration, invariant in y, no B_y
- Tail-like structure, length scale L_x >> L_z. $\varepsilon = \frac{L_z}{L_x} \ll 1$
  Terms of order ε^2 are neglected.
- Slow time dependence, $\partial/\partial t$, v, and E are small (order δ) and terms of order δ^2 are ignored.
- Physically this means that the convection time is much larger as the travel time of waves.
  => Quasistatic evolution
- Constant temperature and particle conservation.
- Inner (dipolar part) of magnetosphere is not included in model (Has been done later, Becker et al. 2001)

How well are these assumptions fulfilled?

Convection time scale	t_c = 1 hour
Convection velocity	v_c = 50 km/s
MHD phase velocity	v_{MHD} = 500 km/s
Thermal ion velocity	v_T = 500 km/s
Characteristic length for variation along the tail	L_x = 50 R_E
Lobe magnetic field	B_o = 20 γ

We find a wave travel time of $t_{MHD} = L_x/v_{MHD}$ = 640 s and a typical convection electric field of $E_c = v_c B_o$ = 1 mV/m. For the relevant quadratic terms we obtain

$$\left(\frac{t_{MHD}}{t_c}\right)^2 = 0.03$$

$$\left(\frac{v_c}{v_{MHD}}\right)^2 = \left(\frac{v_c}{v_T}\right)^2 = 0.01$$

$$\left(\frac{E}{v_T B_o}\right)^2 = 0.01$$

Source: Schindler & Birn, JGR 1982

$$-\nabla p + \underline{j} \times \underline{B} = 0 \quad \text{Force-balance}$$

$$\underline{A} = (0, A(x,z,t), 0) \qquad \phi = \phi(x,z,t)$$

$$\underline{B} = \nabla A \times \underline{e}_y \qquad p(x,z,t) = p(A(x,z,t),t)$$

$$\underline{j} = \partial p(A,t)/\partial A \, \underline{e}_y \qquad \rho = \frac{m_i N(A)}{V(A,t)}$$

$$-\Delta A(\underline{r},t) = \mu_o \frac{\partial p(A,t)}{\partial A}$$

Grad-Shafranov equation, but now time-dependent.

$$\frac{\partial A}{\partial t} + \underline{v} \cdot \nabla A = \frac{dA}{dt} = 0 \quad \text{Ideal Ohms Law}$$

$$\frac{d}{dt}[p(A,t)^{1/\gamma} V(A,t)] = 0$$

Equation of state, Adiabatic convection

$$V(A,t) = \int_A \frac{ds}{|\nabla A|}$$

Differential flux volume can be computed by differentiation along magnetic fieldlines.

For the stretched tail geometry the Grad-Shafranov-Eq. becomes 1D (in z), but depends parametrically on x and t.

$$\frac{1}{2\mu_o}\left(\frac{\partial A}{\partial z}\right)^2 + p(A,t) = p_o(x,t)$$

$$p_0(x,t) = \frac{B_0^2(x,t)}{2} \quad \text{Boundary condition}$$

The magnetic field in the lobe we get from observations:

$$B_0(x,t) = \sqrt{2}(1+t)\left(1+\Lambda(1+t)^2 x\right)^{-\frac{n}{2}}$$

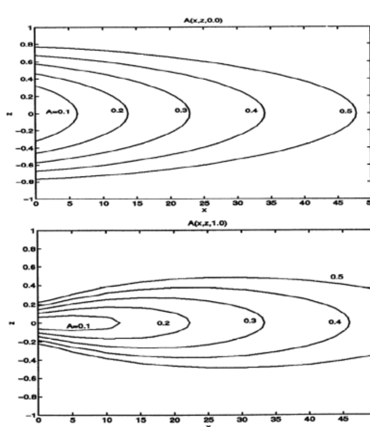
The equation of state can now be written as an integral:

$$\frac{d}{dt}[p(A,t)^{1/\gamma} V(A,t)] = 0$$

$$p(A,t)^{\frac{1}{\gamma}} \int_{p(A,t)}^{p_0(0,t)} \frac{dp_0}{-\frac{\partial p_0(x,t)}{\partial x}\sqrt{p_0 - p(A,t)}} = M(A)$$

As intial pressure profile we use $p(A,0) = \exp(-2A)$ and sove the equation of state selfconsistently with the Grad-Shafranov equation (usually numerically).

$$\frac{1}{2\mu_o}\left(\frac{\partial A}{\partial z}\right)^2 + p(A,t) = p_o(x,t)$$

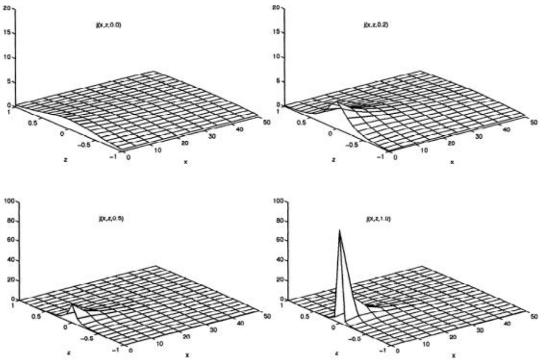


Solution of these equations

Changing of the magnetic field in the magnetotail. Sequence of equilibria constraint by ideal MHD.

Source: Wiegelmann & Schindler 1995

Formation of thin current sheets



Formation of thin current sheets

- Within our model assumptions strong current concentrations (thin current sheets) form.
- Such configurations are prone to current driven micro-instabilities (kinetic instabilities).
- Micro-instabilities cause resistivity on macroscopic MHD-scales.
- The assumption of ideal MHD is violated and we need (at least) use resistive MHD for further investigations.
- => Topology changes by magnetic reconnection and a fast dynamic evolution is possible (not quasistatic anymore) => Eruption phase of substorms start.

Resistive magnetotail MHD-simulations

(Otto et al. 1990, JGR)

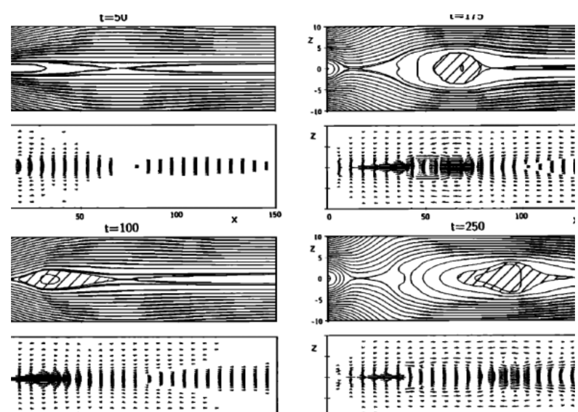
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} = -\nabla \cdot \left(\rho \mathbf{v}\mathbf{v} + \frac{1}{2}\left(p + \mathbf{b}^2\right) \underline{\mathbf{1}} - \mathbf{b}\mathbf{b} \right)$$

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{b} - \eta \mathbf{j})$$

$$\frac{\partial h}{\partial t} = -\nabla \cdot (h \mathbf{v}) + \frac{\gamma - 1}{\gamma} h^{1-\gamma} \eta \mathbf{j}^2$$

$$\mathbf{j} = \nabla \times \mathbf{b}$$



Source: Otto et al. 1990, J

Thin current sheets

- We can understand the formation of strong current concentrations within MHD.
- Spatial scales in thin current sheets become very small and comparable with ion gyro-radius (about 500 km in magnetotail).
- => A basic assumption of MHD is violated and we need to apply kinetic theory (Vlasov equation).
- Stationary Vlasov equilibria.
- Time dependent evolution (either solve Vlasov-eq. directly or indirectly by particle in cell simulations.)

Vlasov-Maxwell Equations

kinetic description of a collisionless plasma

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \sum_\alpha \overline{n}_\alpha q_\alpha \int f_\alpha \, d\mathbf{v} + \frac{\rho_{q\,\mathrm{ext}}}{\epsilon_0}$$

$$\nabla \times \mathbf{B} = \mu_0 \sum_\alpha \overline{n}_\alpha q_\alpha \int \mathbf{v} \, f_\alpha \, d\mathbf{v} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}_{\mathrm{ext}}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

α stands for ions and electrons.

Properties of Vlasov Equation

- Vlasov Equation conserves particles
- Distribution functions remains positive
- Vlasov equation has many equilibrium solutions

$$\mathbf{v} \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{v}} = 0$$

Index 0 stands for equilibrium. Any distribution function $f_{\alpha 0}$ which depends only on constants of motions (say $a(x,v)$, $b(x,v)$, ...) of the particle trajectories solves the stationary Vlasov equation. => Proof see Lecture 4, Kinetic Theory

$$f_{\alpha 0}(x,v) = f_{\alpha 0}(a(x,v), b(x,v), \dots)$$

Thin current sheets, Kinetic theory

Source: Schindler&Birn, JGR 2002

- Constants of motion (2D, x,z):
  - momentum in y $P_y = mv_y + qA(x,z)$
  - Hamiltonian $H = \frac{1}{2m}(P_x^2 + P_z^2) + \frac{1}{2m}[P_y - qA(x,z)]^2 + q\phi(x,z)$
- Stationary solution of Vlasov equation: $f_1 = F(H, P_y)$
- Macroscopic quantities (charge and current density) we get by integration over velocity space:

$$\sigma(A,\phi) = \sum_s q_s n_s = \sum_s q_s \int F_s(H_s, P_{ys}) d^3v$$

$$j_y(A,\phi) = \sum_s q_s \int v_y F_s(H_s, P_{ys}) d^3v$$

Thin current sheets, Kinetic theory

- Inserting into Maxwell equations: $-\Delta A = \mu_0 j_y(A,\phi)$
- Using the quasi-neutrality condition $\sigma(A,\phi) = 0$ we can eliminate the electro-static potential and derive the Grad-Shafranov equation (like in MHS):

$$-\Delta A = \mu_0 j_y(A)$$

- Plasma pressure is isotropic (in x,z) for these form of distribution functions:

$$p(A,\phi) = \sum_s \int \frac{m_s}{2}(v_x^2 + v_z^2) F_s(H_s, P_{ys}) d^3v \qquad j_y = \frac{\partial p}{\partial A}$$

Thin current sheets, Kinetic theory

- Remember that any distribution function depending only on constants of motions solves the stationary Vlasov equation. Examles used in (Schindler & Birn 2002):
- LTE => Local Maxwellian of Hamiltonian:

$$F_s(H, P_y) = C_s \exp\left(-\frac{H_s}{k_B T_s}\right) g_s(P_y)$$

- The momentum in y causes electric currents (ions and electrons move in opposite direction, but the currents in the same. P_y profile can be derived from the result of sequences of magneto-static equlibria.
- Please note that this still leads to some freedom in the distribution functions, e.g. different temperatures for ions and electrons, thickness of sheet can be used.

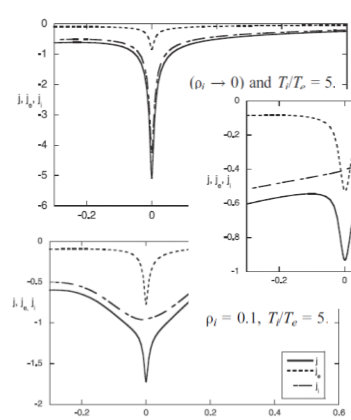
Thin current sheets, Kinetic theory

- Dimensionless parameters:

$$\tau_e = \frac{T_e}{T}, \quad \tau_i = \frac{T_i}{T}, \quad \rho_i = \frac{\sqrt{m_i k_B T_i}}{eB_0 L}, \quad \rho_e = \frac{\sqrt{m_e k_B T_e}}{eB_0 L},$$

Ratio of ion-gyro-radius to spatial dimension L of the current sheet.

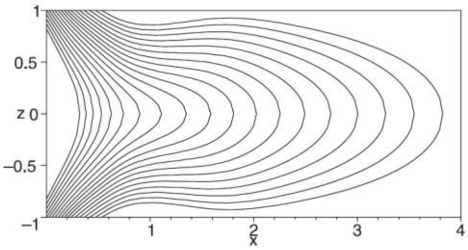
- Remember: MHD is valid only if spatial dimension L is much larger as the gyroradius. As larger this ratio, as more important are kinetic effects.



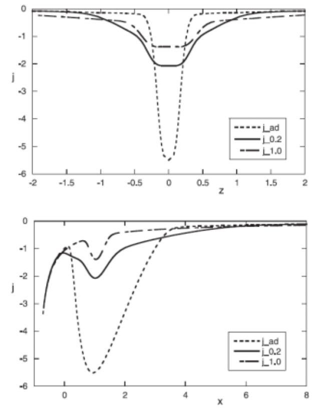
Pictures from Schindler & Birn, JGR 2002

Electron current becomes more important if spatial dimensions comparable with ion-gyro-radius

Thin current sheets, Kinetic theory



As in MHD, A(x,z) defines the magnetic field lines and the relation A-> J_y can be used to compute J_y(x,z). (Picture from Schindler & Birn, JGR 2002)



Thin current sheets, Kinetic theory

- j_ad corresponds to MHD case (gyro-radius infinitesimal small)
- Taking finite gyroradius effects into account localises the sheet in x, but smears it out in z.
- Ouside the current sheet, MHD is valid and the configurations are the same.

Pictures from Schindler & Birn, JGR 2002

Magnetosphere

- We can understand the growth phase of a substorm as a sequence of magneto-static equilibria.
- Interaction with the solar wind leads to stretching of the magnetotail and formation of thin current sheets in the center of the plasma-sheet.
- Thin current sheets cause resistivity on MHD-scales and lead to magnetic reconnection and initiation of the dynamic phase of a substorm.
- This eruptive phase can be simulated with time-dependent, resistive MHD-simulations.
- In the thin current sheets itself the MHD-approach is not valid and we need to apply a kinetic model.