Space Plasma Physics

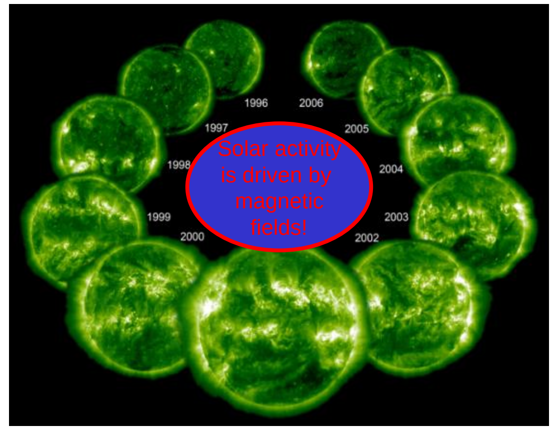
Thomas Wiegelmann, 2012

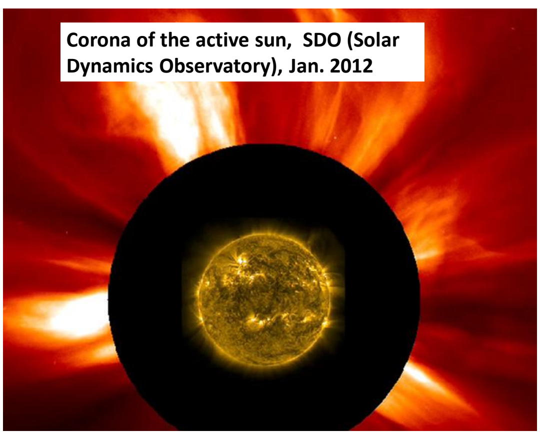
Physical Processes

8. Plasma Waves, instabilities and shocks
9. Magnetic Reconnection

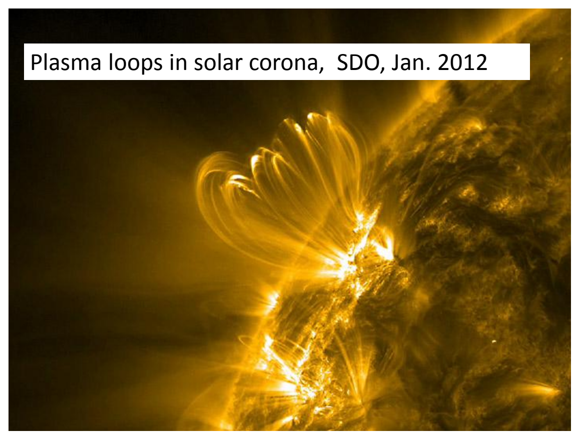
Applications

10. Planetary Magnetospheres
11. Solar activity
12. Transport Processes in Plasmas





Corona of the active sun, SDO (Solar Dynamics Observatory), Jan. 2012



Plasma loops in solar corona, SDO, Jan. 2012

Solar Eruptions

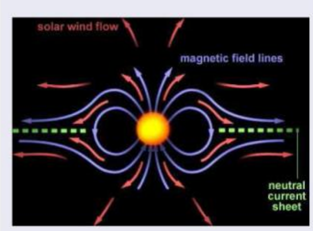
- The solar coronal plasma is frozen into the coronal magnetic field and plasma outlines the magnetic field lines.
- Coronal configurations are most of the time quasistatic and change only slowly.
- Occasionally the configurations become unstable and develop dynamically fast in time, e.g., in coronal mass ejections and flares.

Global magnetic field

Shape

- basic shape: dipole field $\sim 10^{-4}T$
- extension: IMF, heliosphere
- superimposed: complex series of local fields $\sim 0.1T$

Interplanetary magnetic field



Encyclopedia of Science.

How to model the stationary Corona?

$$\underbrace{(\nabla \times \mathbf{B}) \times \mathbf{B}}_{\text{Lorentz force}} - \underbrace{\cancel{\mu_0 \nabla p}}_{\text{pressure gradient}} - \underbrace{\cancel{\mu_0 \rho \nabla \Psi}}_{\text{gravity}} = \mathbf{0}$$

Low plasma Beta in corona → Neglect plasma pressure+gravity → Force-free Fields

Force-Free Fields

$$\begin{aligned} (\nabla \times \mathbf{B}) \times \mathbf{B} &= \mathbf{0} \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \quad \xleftrightarrow{\text{Equivalent}} \quad \begin{aligned} \nabla \times \mathbf{B} &= \alpha \mathbf{B} \\ \mathbf{B} \cdot \nabla \alpha &= 0 \end{aligned}$$

Relation between currents and magnetic field. Force-free functions is constant along field lines, but varies between field lines.
=> nonlinear force-free fields

Further simplifications

- Potential Fields (no currents)
- Linear force-free fields
(currents globally proportional to B-field)

Potential and Linear Force-Free Fields

We have to solve the equations:

$$(\nabla \times \mathbf{B}) = \alpha \mathbf{B}$$
$$\nabla \cdot \mathbf{B} = 0$$

Here: global constant linear force-free parameter

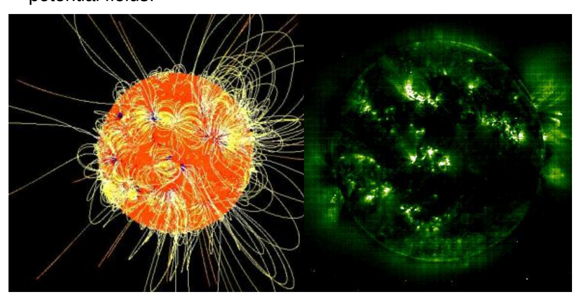
We take the curl of the first equation and apply a vector identity:

$$(\Delta + \alpha^2)\, \mathbf{B} = 0 \quad \text{(Helmholtz equation)}$$

We solve $(\Delta + \alpha^2) B_z = 0$ with observed (e.g. MDI) $B_z(x, y, 0)$ on the photosphere and get B_x and B_y from the other components of the Helmholtz equation and $\nabla \cdot \vec{B} = 0$.

A subclass of force-free fields are current-free potential fields.

$$\begin{aligned} \nabla \times \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \mathbf{B} &= \nabla \Phi \\ \Delta \Phi &= 0 \end{aligned}$$



Global potential Field — Coronal Plasma seen in EUV.

Nonlinear Force-Free Fields

$$\begin{aligned} (\nabla \times \mathbf{B}) \times \mathbf{B} &= \mathbf{0} \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \quad \xleftrightarrow{\text{Equivalent}} \quad \begin{aligned} \nabla \times \mathbf{B} &= \alpha \mathbf{B} \\ \mathbf{B} \cdot \nabla \alpha &= 0 \end{aligned}$$

- For equilibria with symmetry we can reduce the force-free equations to a Grad-Shafranov equation (Low&Lou 1990).
- In 3D, the NLFFF-equations are solved numerically.
- Suitable boundary conditions are derived from measurements of the photospheric field vector.
  - B_n and J_n for positive or negative polarity on boundary (**Grad-Rubin**)
  - Magnetic field vector $B_x\, B_y\, B_z$ on boundary (**Magnetofrictional, Optimization**)

Nonlinear force-free fields in 2.5D

(Source: Low&Lou, ApJ 1990)

$$\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$$
$$\boldsymbol{B} \cdot \nabla \alpha = 0$$

- In spherical geometry (invariant in phi, but B-field has 3 components) the magnetic field can be represented as:

$$\boldsymbol{B} = \frac{1}{r \sin\theta}\left(\frac{1}{r}\frac{\partial A}{\partial \theta}\hat{\boldsymbol{r}} - \frac{\partial A}{\partial r}\hat{\boldsymbol{\theta}} + Q\hat{\boldsymbol{\phi}}\right) \qquad \alpha = \frac{dQ}{dA}$$

- The force-free equations reduce to a Grad-Shafranov Eq.:

$$\frac{\partial^2 A}{\partial r^2} + \frac{1-\mu^2}{r^2}\frac{\partial^2 A}{\partial \mu^2} + Q\frac{dQ}{dA} = 0 \qquad \mu = \cos\theta$$

- Low&Lou 1990 found separable solutions:

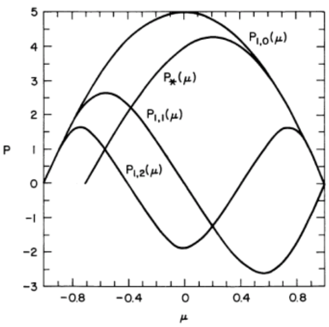
$$A = \frac{P(\mu)}{r^n}, \qquad (1-\mu^2)\frac{d^2P}{d\mu^2} + n(n+1)P + a^2\,\frac{1+n}{n}\,P^{1+2/n} = 0$$

$$Q(A) = aA^{1+1/n}$$

a and n are free parameters.

Nonlinear force-free fields in 2.5D

$$(1-\mu^2)\frac{d^2P}{d\mu^2} + n(n+1)P + a^2\,\frac{1+n}{n}\,P^{1+2/n} = 0$$



The angular functions P are computed by solving an ordinary (but nonlinear) differential equation. This can be easily done, e.g. with a Runge-Kutta method.

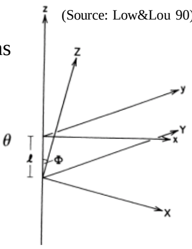
$\Rightarrow$ Nonlinear force-free field in spherical geometry with symmetry in phi.

(Source: Low&Lou 90)

Nonlinear force-free fields in 2.5D
=> 3D looking fields in cartesian coordinates

A nice property of these equilibria is, that one get 3D-looking configurations by shifting and rotating geometry.
=> Model for a solar Active Region.

(Source: Low&Lou 90)



$X = r\sin\theta\cos\phi,\ Y = r\sin\theta\sin\phi,\ Z = r\cos\theta$

$X = x\cos\Phi - (z+l)\sin\Phi$

$Y = y\,,$

$Z = x\sin\Phi + (z+l)\cos\Phi$

$B_X = B_r\sin\theta\cos\phi + B_\theta\cos\theta\cos\phi - B_\phi\sin\phi\,,$

$B_Y = B_r\sin\theta\sin\phi + B_\theta\cos\theta\sin\phi + B_\phi\cos\phi\,,$

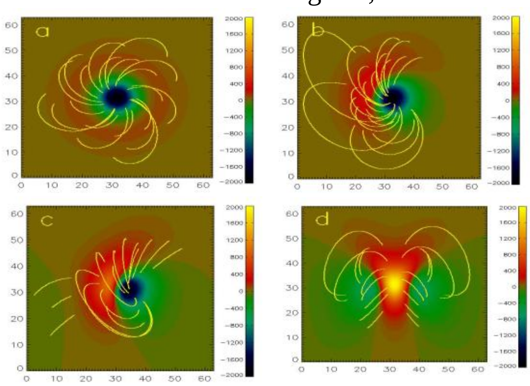
$B_Z = B_r\cos\theta - B_\theta\sin\theta\,,$

$B_x = B_X\cos\Phi + B_Z\sin\Phi\,,$

$B_y = B_Y\,,$

$B_z = -B_X\sin\Phi + B_Z\cos\Phi\,.$

Low&Lou-model active regions, PHI was varied



Some properties for force-free fields in 3D
(Molodensky 1969, Aly 1989)

$\int_V \nabla\cdot\mathbf{B}\,d^3x = 0 \Rightarrow \oint_S \mathbf{B}\,d\mathbf{S} = 0$ Boundary flux balanced

$\int_V (\nabla\times\mathbf{B})\times\mathbf{B}\,d^3x = 0$

$\int_V \nabla\cdot T\,d^3x = 0 \Rightarrow \oint_S T d\mathbf{S} = 0$

$T_{ij} = B_iB_j - \frac{1}{2}\mathbf{B}^2\delta_{ij}$ Maxwell Stress torque

No net force on boundaries

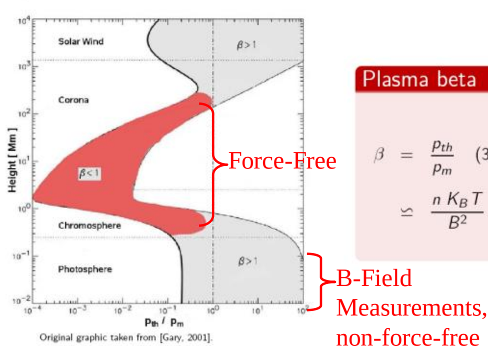
$\int_V \mathrm{r}\times[(\nabla\times\mathbf{B})\times\mathbf{B}]\,d^3x = 0$

$\int_V \nabla\cdot\tilde{T}\,d^3x = 0 \Rightarrow \oint_S \tilde{T}d\mathbf{S} = 0$

$\tilde{T}_{ij} = \epsilon_{jkl}\,r_k\,T_{ij}$

No net torque on boundaries

Extrapolation domain



Original graphic taken from [Gary, 2001].

Plasma beta

$$\beta = \frac{p_{th}}{p_m} \quad (3)$$
$$\simeq \frac{n\,K_B\,T}{B^2}$$

MHD-relaxation
Chodura & Schlueter 1981

$$\nu\mathbf{v} = (\nabla\times\mathbf{B})\times\mathbf{B}$$
$$\mathbf{E}+\mathbf{v}\times\mathbf{B} = 0$$
$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$
$$\nabla\cdot\mathbf{B} = 0$$
$$\nu = \frac{1}{\mu}|\mathbf{B}|^2$$

The equations are combined to

$$\boxed{\frac{\partial\mathbf{B}}{\partial t}} = \mu\,\mathbf{F}_{\mathrm{MHD}}$$

$$\mathbf{F}_{\mathrm{MHD}} = \nabla\times\left(\frac{[(\nabla\times\mathbf{B})\times\mathbf{B}]\times\mathbf{B}}{B^2}\right)$$

Optimization
Wheatland et al. 2000

$$L = \int_V \left[B^{-2}|(\nabla\times\mathbf{B})\times\mathbf{B}|^2 + |\nabla\cdot\mathbf{B}|^2\right]\,d^3V$$

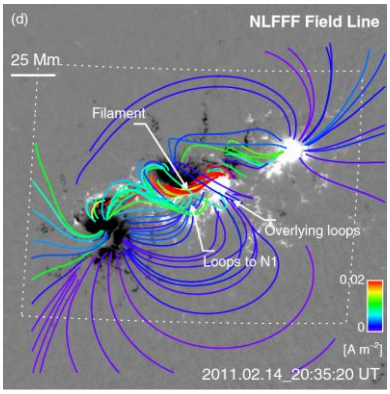
Take functional derivative:

$$\Rightarrow \frac{1}{2}\frac{dL}{dt} = -\int_V \frac{\partial\mathbf{B}}{\partial t}\cdot\tilde{\mathbf{F}}\,d^3x - \int_S \frac{\partial\mathbf{B}}{\partial t}\cdot\tilde{\mathbf{G}}\,d^2x$$

$$\boxed{\frac{\partial\mathbf{B}}{\partial t}} = \mu\,\mathbf{F}$$

$$\mathbf{F} = \nabla\times\left(\frac{[(\nabla\times\mathbf{B})\times\mathbf{B}]\times\mathbf{B}}{B^2}\right) + \Big\{-\nabla\times\left(\frac{((\nabla\cdot\mathbf{B})\,\mathbf{B})\times\mathbf{B}}{B^2}\right) - \mathbf{\Omega}\times(\nabla\times\mathbf{B}) - \nabla(\mathbf{\Omega}\cdot\mathbf{B}) + \mathbf{\Omega}(\nabla\cdot\mathbf{B}) + \mathbf{\Omega}^2\,\mathbf{B}\Big\}$$

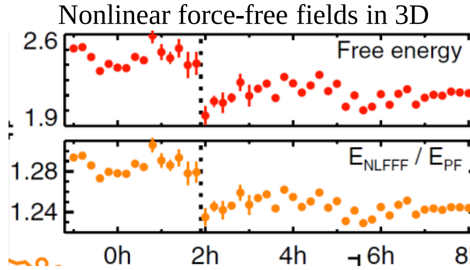
$$\Omega = B^{-2}\left[(\nabla\times\mathbf{B})\times\mathbf{B} - (\nabla\cdot\mathbf{B})\,\mathbf{B}\right]$$



Nonlinear force-free coronal magnetic fields extrapolated with optimization method from SDO measurements. Selected field lines are colour coded by the amount of electric current they contain. 5h after this snapshot a X-class (large!) Flare occured.

Source:Sun et al., ApJ 2012

Nonlinear force-free fields in 3D



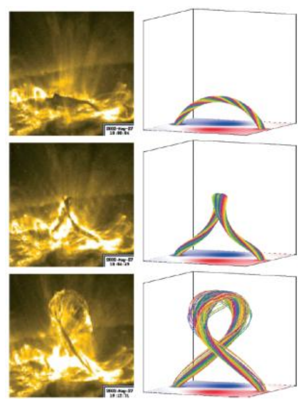
Monitoring of free magnetic energy before and after the X-class Flare (dotted vertical line) by a sequence of force-free equilibria. During the Flare magnetic energy is converted to thermal and kinetic energy => Dynamic phase, cannot be modelled by sequences of equilibria. Source:Sun et al., 2012

Time dependent evolution of force-free fields

In the limit of a vanishing plasma beta the ideal, time dependent MHD-equations are:

$$
\begin{aligned}
\partial_t \rho &= -\nabla\cdot(\rho\,\mathbf{u})\,,\\
\rho\,\partial_t \mathbf{u} &= -\rho\,(\mathbf{u}\cdot\nabla)\,\mathbf{u} + \mathbf{j}\times\mathbf{B}\,,\\
\partial_t \mathbf{B} &= \nabla\times(\mathbf{u}\times\mathbf{B})\,,\\
\mathbf{j} &= \mu_0^{-1}\nabla\times\mathbf{B}\,.
\end{aligned}
$$

Nonlinear force-free equilibria contain free energy and can become unstable, e.g. an ideal kink-instability.
Source: Kliem and Török, ESASP 2004



Time dependent evolution of force-free fields

Left: Confined filament eruption observed with Trace in May 2002.
Right: Magnetic field lines of a kink-unstable flux robe.

Source: Kliem&Török, ApJL 2005

Movies: http://www.lesia.obspm.fr/perso/tibor-torok/res.html

Coronal plasma and magnetic field

$$\mathbf{B}\bullet\ \underbrace{(\nabla\times\mathbf{B})\times\mathbf{B}}_{\text{Lorentz force}} - \underbrace{\mu_0\nabla p}_{\text{pressure gradient}} - \underbrace{\mu_0\,\rho\,\nabla\Psi}_{\text{gravity}} = \mathbf{0}$$

for vertical scales < 0.1 R_S (gravitational scale height) =>
Pressure constant on field lines

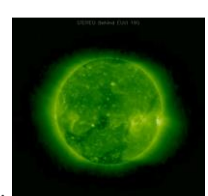
$$\boxed{\mathbf{B}\cdot[\nabla p} + \cancel{\rho\nabla\Psi}] = 0$$

Force-free equilibria and coronal plasma

$$\underbrace{\mathbf{j}\times\mathbf{B}}_{\text{small}} = \underbrace{\nabla p + \rho\nabla\Psi}_{\text{small}}$$

$$
\begin{aligned}
\beta &\ll 1, \quad \text{but} \quad \beta \neq 0\\
|\mathbf{j}\times\mathbf{B}| &\ll |\mathbf{j}|\,|\mathbf{B}|\\
j_\perp &\ll j_\parallel
\end{aligned}
$$

$j_\perp$ structures the plasma.



Coronal plasma loop modelling

- The magnetic field structures the solar corona and due to the low plasma-beta we can neglect plasma effects when modelling the coronal magnetic field.
- Modeling the coronal plasma can be done with 1D-hydrodynamics models along the magnetic field lines. (Question/Exercise: Why not MHD along loops?)
- Time-dependent processes (like reconnection) are thought to be important for heating (and flares, eruptions etc.)
- Here we concentrate on stationary solution.

1D plasma loop modelling

Source: Aschwanden&Schrijver, ApJ 2002

$$\frac{1}{A}\frac{d}{ds}(nvA) = 0$$ Continuity Equation

$$mnv\frac{dv}{ds} = -\frac{dp}{ds} + \frac{dp_{\rm grav}}{dr}\left(\frac{dr}{ds}\right)$$ Momentum Eq.

$$\frac{1}{A}\frac{d}{ds}(nvA[\epsilon_{\rm enth} + \epsilon_{\rm kin} + \epsilon_{\rm grav}] + \underbrace{AF_C}_{\text{conduction}}) = \underbrace{E_H}_{\text{heating}} + \underbrace{E_R}_{\text{radiative cooling}}$$

Energy Eq.

A is the loop cross section and s a coordinate along the loop.

$$\epsilon_{\rm enth}(s) = \tfrac{5}{2}k_{\rm B}T(s) \qquad \epsilon_{\rm kin}(s) = \tfrac{1}{2}mv^2(s)$$

$$\epsilon_{\rm grav}(r) = -\frac{GM_\odot m}{r} = -mg_\odot\left(\frac{R_\odot^2}{r}\right)$$

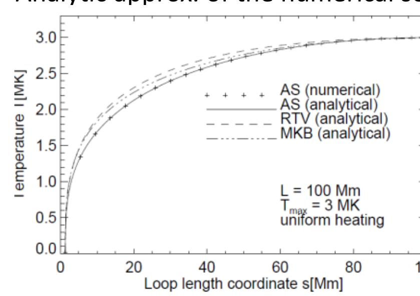
$$F_C(s) = \left[-\kappa T^{5/2}(s)\frac{dT(s)}{ds}\right]$$ conductive flux

$$E_H(s) = E_0 \exp\left(-\frac{s}{s_H}\right)$$ heating rate, not very well known, E_0 is heating at base

$$E_R(s) = -n_e^2(s)\Lambda[T(s)]$$ radiative cooling, proportional to square of electron density and a factor (Temperature dependent, not well known)

Source: Aschwanden&Schrijver 2002

Analytic approx. of the numerical solution



$$T(s) = T_{\rm max}\left[1 - \left(\frac{L-s}{L-s_0}\right)^a\right]^b$$

Computing the numerical solution is expensive and analytic approximations useful. (L is loop half-length, a and b are fitting parameters.)

Source: Aschwanden&Schrijver 2002

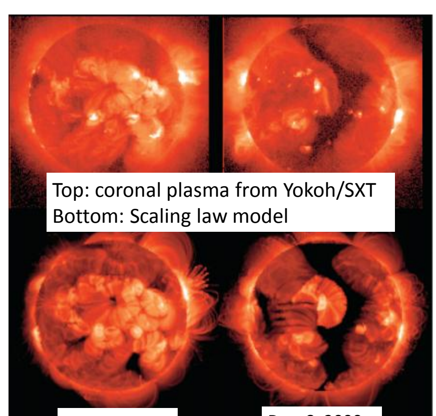
Scaling laws

- Analytic expression help to compute the plasma along many field lines, which is numerically very expensive. Plasma pressure can be derived by integration.
- Analytic models are simplifications, based on circular loops and static equilibria without flow.
- In a subsequent work Schrijver et al. 2005 derived scaling laws and related the heating flux density to magnetic field strength at loop base B_base and loop half length L:

$$F_H = \alpha B_{\rm base}^{\beta} L_{\rm half}^{\lambda} f(B_{\rm base})$$

- Best agreement was found (in CGS-units) for:

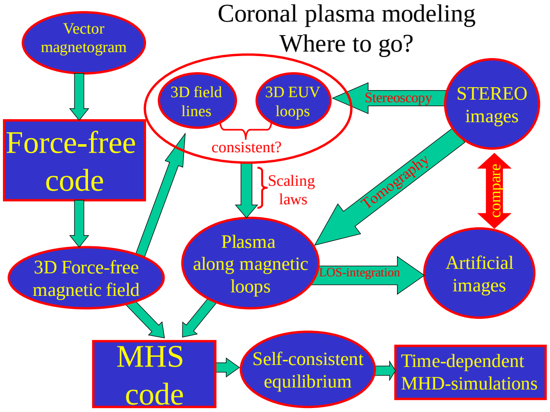
$$F_H \approx 4\times 10^{14} B^{1.0\pm0.3} / L^{1.0\pm0.5}$$



The coronal plasma model in the bottom images is based on a potential field model for the magnetic field and scaling laws for the plasma.

(Source: Schrijver et al. 2005)

Coronal plasma modeling
Where to go?



Summary: coronal plasma

- Due to the low plasma-beta in the solar corona the coronal magnetic field is force-free.
- With nonlinear force-free models we can monitor the total and free energy of active regions.
- Configurations with free energy can become unstable and erupt => Flares, mass ejections.
- Eruptive phase can be modelled with time-dependent MHD-simulations.
- Coronal plasma is frozen into the magnetic field and can be modelled by 1D hydrodynamics along the magnetic field lines.