Space Plasma Physics

Thomas Wiegelmann, 2012

1. Basic Plasma Physics concepts
2. Overview about solar system plasmas

Plasma Models

3. Single particle motion, Test particle model
4. Statistic description of plasma, BBGKY-Hierarchy and kinetic equations
5. Fluid models, Magneto-Hydro-Dynamics
6. Magneto-Hydro-Statics
7. Stationary MHD and Sequences of Equilibria

Plasma models

- **Test particles:**
  Study motion of individual charged particles under the influence of external electro-magnetic (EM) fields
- **Kinetic models:**
  Statistic description of location and velocity of particles and their interaction + EM-fields. (Vlasov-equation, Fokker-Planck eq.)
- **Fluid models:**
  Study macroscopic quantities like density, pressure, flow-velocity etc. + EM-fields (MHD + multifluid models)
- **Hybrid Models:** Combine kinetic + fluid models

Single particle motion, Test particle model

- What is a test particle?
- Charged particles homogenous magnetic fields => Gyration
- Charged particles in inhomogenous fields => Drifts
- Adiabatic invariants
- Magnetic mirror and radiation belts

Test particles:

In the test-particle approach the charged particles move under the influence of electric and magnetic fields.
Back-reaction of the particles is ignored => model is not self-consistent.

Equation of Motion, Lorentz-force:

$$\mathbf{v}(\mathbf{t}) = \frac{\partial \mathbf{r}(\mathbf{t})}{dt}$$

$$m\frac{\partial \mathbf{v}(\mathbf{t})}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Test particles, special cases

(calculate on blackboard + exercises)

- Static homogenous electric field, no B-field
- Static homgeneous magnetic field => Gyration, magnetic moment
- Static, hom. electromagnetic fields (exercise)
- Static inhomogeneous B-field.
- Homogeneous, time-varying electromo-magnetic fields (exercise).
- Generic cases of time-varying inhomogenous EM-fields => Treat numerically.

Particles in magnetic field

Kinetic energy is a constant of motion in B-Fields

$$\frac{d}{dt}\left(\frac{m}{2}v^2\right) = q\mathbf{v} \cdot (\mathbf{v} \times \mathbf{B}) = 0$$

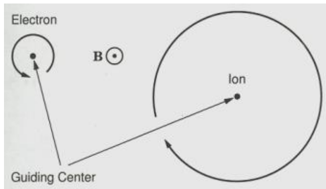
In B-Fields particles (Electrons, Protons) gyrate

Larmor (or gyro) frequency $$w_g = \frac{|q|B}{m}$$

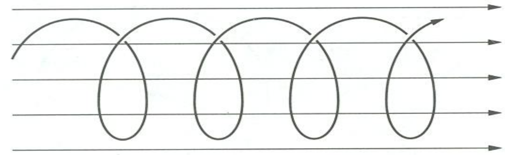
Larmor radius $$r_g = \frac{v_\perp m}{B|q|}$$

Gyration of ions and electrons

$$x - x_0 = r_g \sin \omega_g t$$
$$y - y_0 = r_g \cos \omega_g t$$



The equation describes a circular orbit around the field with gyroradius, r_g, and gyrofrequency, ω_g. The orbit's center (x_0,y_0) is called the *guiding center*. The gyration represents a microcurrent, which creates a field opposite to the background one. This behaviour is called *diamagnetic effect*.

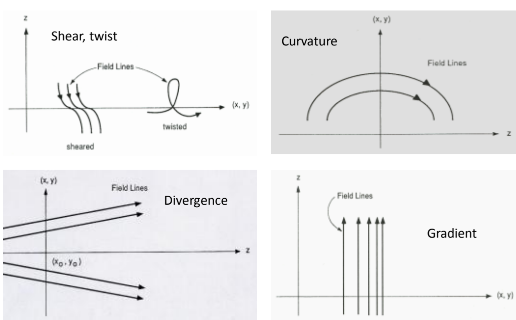


$$r_g = \frac{v_\perp}{\mid \omega_g \mid} = \frac{m v_\perp}{\mid q \mid B}$$

$$\alpha = \tan^{-1}\left(\frac{v_\perp}{v_\parallel}\right)$$

If one includes a constant speed parallel to the field, the particle motion is three-dimensional and looks like a *helix*. The *pitch angle* of the helix or particle velocity with respect to the field depends on the ratio of perpendicular to parallel velocity components.

Nonuniform magnetic fields in space



Particles in magnetic field

- Concept of gyrating particles remains useful for inhomogeneous and time-dependent magnetic fields => Drifts, Valid if:
- Spatial scales for B-field changes are large compared to Larmor radius.
- Electric fields, gravity, magnetic field curvature etc. also cause drifts
(some we will study in excercises)

Inhomogeneous B-fields

- Spatial scales for B-field changes are large compared to Gyro radius.

$$\epsilon = \left| \frac{\mathbf{v} \cdot \nabla \mathbf{B}}{w_g \mathbf{B}} \right| \ll 1$$

- Magnetic moment remains constant in weakly inhomogenous B fields => Adiabatic Invariant

$$\mu = \frac{m v_\perp^2}{2B}$$

Adiabatic invariants of motion

- We have a quantity which is small, like

$$\epsilon = \left| \frac{\mathbf{v} \cdot \nabla \mathbf{B}}{w_g \mathbf{B}} \right| \ll 1$$

- An adiabatic invariant stays constant over a period of the order $1/\epsilon$, while the electro-magnetic field changes of the order O(1) in the same period.
- What happens if $\epsilon \rightarrow 0$?
- Adiabatic invariant becomes constant of motion.

Magnetic mirror

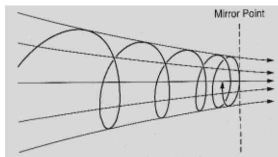
Let us follow the guiding center of a particle moving along an inhomogeneous magnetic field by considering the magnetic moment:

$$\mu = \frac{mv^2 \sin^2 \alpha}{2B}$$

where we used the *pitch angle* α. Apparently, pitch angles at different locations are related by the corresponding magnetic field strengths:

$$\frac{\sin^2 \alpha_2}{\sin^2 \alpha_1} = \frac{B_2}{B_1}$$

The point where the angle reaches 90° is called the *mirror point*.



Adiabatic invariants of motion

In classical Hamiltonian mechanics the action integral

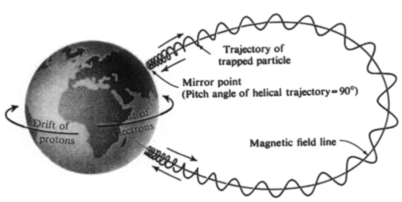
$$J_i = \oint p_i dq_i$$

is an invariant of motion for any change that is slow as compared to the oscillation frequency associated with that motion.

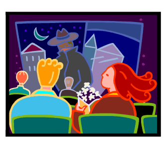
- *Bounce motion* between mirror points $J = \oint mv_{\|} ds$
- *Drift motion* in azimuthal direction, with planetary magnetic moment *M* $\Phi = \frac{2\pi m}{q^2} M = \text{const}$

Magnetic flux, $\Phi_\mu = B\pi r_g^2$, through surface encircled by the gyro orbit is constant.

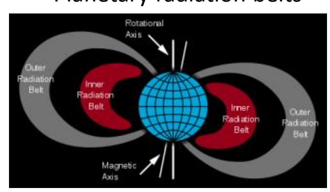
Application: Van Allen Belt



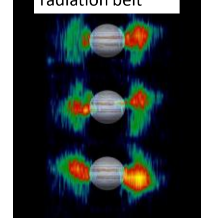
A *dipole magnetic field* has a field strength minumum at the equator and converging field lines at the polar regions (mirrors). Particles can be *trapped* in such a field. *They perform gyro, bounce and drift motions*.

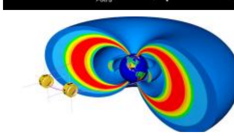


Application: Van Allen Belt, Planetary radiation belts



Jupiter's variable radiation belt





Radiation Belt Storm Probes (RBSP), launch 2012(?)

Motion of particles in radiation belts can be modelled with Test-particle approach.

Magnetic drifts

Inhomogeneity will lead to a drift. A typical magnetic field in space will have gradients, and thus field lines will be curved. We Taylor expand the field:

$$\mathbf{B} = \mathbf{B}_0 + (\mathbf{r} \cdot \nabla)\mathbf{B}_0$$

where B_0 is measured at the guiding center and r is the distance from it. Modified equation of motion:

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B}_0) + q\left[\mathbf{v} \times (\mathbf{r} \cdot \nabla)\mathbf{B}_0\right]$$

Expanding the velocity in the small drift plus gyromotion, $v = v_g + v_\nabla$, then we find the stationary drift:

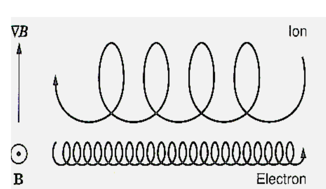
$$\mathbf{v}_\nabla = \frac{1}{B_0^2}\langle(\mathbf{v}_g \times (\mathbf{r} \cdot \nabla)\mathbf{B}_0) \times \mathbf{B}_0\rangle$$

Magnetic drifts

We time average over a gyroperiod and obtain:

$$\mathbf{v}_\nabla = \frac{mv_\perp^2}{2qB^3}(\mathbf{B} \times \nabla B)$$

The non-uniform magnetic field B leads to a *gradient drift* perpendicular to both, the field and its gradient:



General force drifts

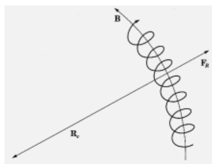
By replacing the electric field *E* in the drift formula by any field exerting a force *F/q*, we obtain the general *guiding-center drift*:

$$\mathbf{v}_F = \frac{1}{\omega_g}\left(\frac{\mathbf{F}}{m} \times \frac{\mathbf{B}}{B}\right)$$

In particular when the field lines are curved, the centrifugal force is

$$\mathbf{F}_R = m v_\parallel^2 \frac{\mathbf{R}_c}{R_c^2}$$

where R_c is the local radius of curvature.



Summary of guiding center drifts

E×B Drift:

Polarization Drift:

Try to calculate these Drifts as excercise

Gradient Drift: $\mathbf{v}_\nabla = \frac{m v_\perp^2}{2q B^3}(\mathbf{B} \times \nabla B)$ $\quad \mathbf{j}_\nabla = \frac{n_e(\mu_i + \mu_e)}{B^2}(\mathbf{B} \times \nabla B)$

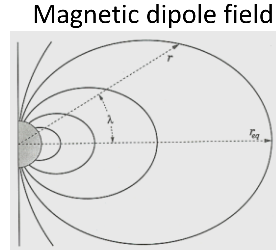
Curvature Drift: $\mathbf{v}_R = \frac{m v_\parallel^2}{q R_c^2 B^2}(\mathbf{R}_c \times \mathbf{B})$ $\quad \mathbf{j}_R = \frac{2n_e(W_{i\parallel} + W_{e\parallel})}{R_c^2 B^2}(\mathbf{R}_c \times \mathbf{B})$

Associated drifts are corresponding drift currents.

Gyrokinetic approach

- We can distinguish the motion of charged particles into gyration of the particle and motion of the gyration center.
- An exact mathematical treatment is possible within Hamilton mechanics by using non-canonical transformation.
- => Guiding-center approximation.
[Outside scope of this Lecture, see Balescue, Transport-Processes in Plasma, Vol. 1, 1988]
- Guiding center approach remains useful concept for selfconsistent kinetic plasma models.

Magnetic dipole field



At distances not too far from the surface the Earth's magnetic field can be approximated by a *dipole field* with a moment:

M_E = 8.05 10^{22} Am²

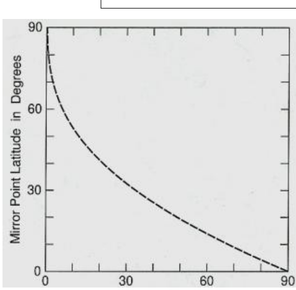
$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{M_E}{r^3}(-2\sin\lambda\hat{\mathbf{e}}_r + \cos\lambda\hat{\mathbf{e}}_\lambda)$$

$$B(\lambda, L) = \frac{B_E}{L^3}\frac{(1 + 3\sin^2\lambda)^{1/2}}{\cos^6\lambda}$$

Measuring the distance in units of the Earth's radius, R_E, and using the equatorial surface field, B_E (= 0.31 G), yields with the so-called *L-shell* parameter ($L = r_{eq}/R_E$) the field strength as a function of latitude, λ, and of *L* as:

Dipole latitudes of mirror points

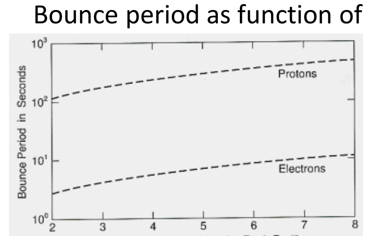
$$\sin^2\alpha_{eq} = \frac{B_{eq}}{B_m} = \frac{\cos^6\lambda_m}{(1 + 3\sin^2\lambda_m)^{1/2}}$$



Equatorial pitch angle in degrees

Latitude of mirror point depends only on pitch angle but not on *L* shell value.

Bounce period as function of *L* shell

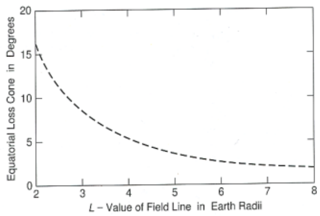


Energy, *W*, is here 1 keV and α_{eq} = 30°.

Bounce period, τ_b, is the time it takes a particle to move back and forth between the two mirror points (*s* is the path length along a given field line).

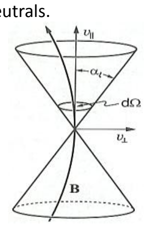
$$\tau_b = 4\int_0^{\lambda_m}\frac{ds}{v_\parallel} = 4\int_0^{\lambda_m}\frac{ds}{d\lambda}\frac{d\lambda}{v_\parallel}$$

Equatorial loss cone for different *L*-values



$$\sin^2 \alpha_\ell = (4L^6 - 3L^5)^{-1/2}$$

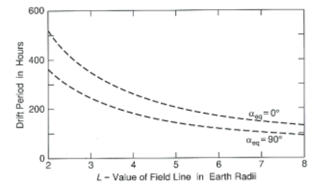
If the mirror point lies too deep in the atmosphere (below 100 km), particles will be absorbed by collisions with neutrals.



The loss-cone width depends only on L but not on the particle mass, charge or energy.

Period of azimuthal magnetic drift motion

Here the energy, *W*, is 1 keV and the pitch angle: α_{eq} = 30° and 90°.



$$\langle \tau_d \rangle \approx \frac{\pi q B_E R_E^2}{3LW}(0.35 + 0.15 \sin \alpha_{eq})^{-1}$$

Drift period is of order of several days. Since the magnetospheric field changes on smaller time scales, it is unlikely that particles complete an undisturbed drift orbit. Radiation belt particles will thus undergo radial (*L*-shell) diffusion!

Summary: Single particle motion

- Gyromotion of ions and electrons arround magnetic field lines.
- Inhomogeneous magnetic fields, electric fields and other forces lead to particle drifts and drift currents.
- Bouncing motion of trapped particles to model radiation belt.
- Constants of motion and adiabatic invariants.
- So far we studied particles in external EM-fields and ignored fields and currents created by the charged particles and collision between particles.