Space Plasma Physics

Thomas Wiegelmann, 2012

Physical Processes

Applications

Transport in neutral gases

- Transport processes in gases have first been studied by Maxwell and Boltzmann, the founders of kinetic theory.
- Aim of transport theory is to understand non-equilibrium processes from first principles.
- For neutral gases this approach was successful in, e.g., pridicting of transport coefficients like:
  - diffusion
  - heat conduction
  - viscosity
  -thermo-diffusion (cross-coefficients for different species)
- Motor for irreversibility are (mainly binary) collisions.

Transport in plasmas

- Theory was driven mainly to understand fully ionized plasmas in fusion experiments (since mid 20th century)
- Naturally scientists applied the well known methods from gas-kinetic-theory to plasmas.
- But there are two major differences

1.) Plasma contains charged particles and (long-ranging) Coulomb interaction dominate interaction of particles.

2.) External electric and magnetic fields act on the charged particle.

Classical transport theory in plasmas

- A formalism (Chapman-Enskog theory for multicomponent gases) treats non-equilibrium gases as (small) perturbations from Maxwell-Boltzmann distribution.
- Incorporating electromagnetic fields, Coulomb-collisions etc. in this formalism lead to the **classical transport theory for plasmas** (Braginskii 1965, Balescu 1988)
- So far so good. But then the theory was compared with experiments and this was really disappointing:
- Classical transport theory grossly underestimated the transport-coefficients, often **by several orders of magnitude!!**

Transport in space-plasmas

- Limitations of classical transport theory apply also to space plasmas.
- Classical theory was succesfull to calculate transport coefficients like conductivity (or it's inverse the resistivity) for weakly ionized plasmas like the ionosphere. (Spitzer resistivity)
- For fully ionized space plasmas we face the same problem as in fusion theory: the transport coefficients (say resistivity) are orders of magnitudes to low.
- From classical theory most space plasmas would be ideal and resistive processes (like reconnection) would not play any role.

Plasma resistivity in partly ionized plasma

In the presence of collisions we have to add a collision term in the equation of motion. Assume collision partners moving at velocity ***u***.

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - m\nu_c(\mathbf{v} - \mathbf{u})$$

In a steady state collisional friction balances electric acceleration. Assume there is no magnetic field, **B = 0**. Then we get for the electrons with ions at rest:

$$\mathbf{E} = -\frac{m_e \nu_c}{e}\mathbf{v}_e$$

Since electrons move with respect to the ions, they carry the current density, **j** = $-en_e\mathbf{v}_e$. Combining this with the above equation yields, **E** = η **j**, with the resistivity:

$$\eta = \frac{m_e \nu_c}{n_e e^2}$$

Conductivity in a magnetized plasma

In a steady state collisional friction balances the Lorentz force. Assume the ions are at rest, $\mathbf{v}_i = \mathbf{0}$. Then we get for the electron bulk velocity:

$$\mathbf{E} + \mathbf{v}_e \times \mathbf{B} = -\frac{m_e \nu_c}{e} \mathbf{v}_e$$

Assume for simplicity that, $\mathbf{B}=B\mathbf{e}_z$. Then we can solve for the electron bulk velocity and obtain the current density, which can in components be written as:

$$\begin{aligned} j_x &= \sigma_0 E_x + \frac{\omega_{ge}}{\nu_c} j_y \\ j_y &= \sigma_0 E_y - \frac{\omega_{ge}}{\nu_c} j_x \\ j_z &= \sigma_0 E_z \end{aligned}$$

$$\sigma_0 = \frac{n_e e^2}{m_e \nu_c}$$

The current can be expressed in the form of Ohm's law in vector notation as: $\mathbf{j} = \boldsymbol{\sigma}\, \mathbf{E}$, with the dyadic conductivity tensor $\boldsymbol{\sigma}$.

Conductivity in a magnetized plasma

For a magnetic field in *z* direction the conductivity tensor $\boldsymbol{\sigma}$ reads:

$$\sigma = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{||} \end{pmatrix}$$

$$\mathbf{j} = \sigma_{||}\mathbf{E}_{||} + \sigma_P \mathbf{E}_{\perp} - \sigma_H (\mathbf{E}_{\perp} \times \mathbf{B})/B$$

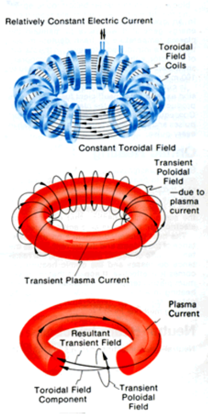
The tensor elements are the *Pedersen*, σ_D, the *Hall*, σ_H, and the *parallel* conductivity. In a weak magnetic field the Hall conductivity is small and the tensor diagonal, i.e. the current is then directed along the electric field.

$$\begin{aligned} \sigma_P &= \frac{\nu_c^2}{\nu_c^2 + \omega_{ge}^2}\sigma_0 \\ \sigma_H &= -\frac{\omega_{ge}\nu_c}{\nu_c^2 + \omega_{ge}^2}\sigma_0 \\ \sigma_{||} &= \sigma_0 = \frac{n_e e^2}{m_e \nu_c} \end{aligned}$$

Successful method for weakly ionized plasmas like inosphere

Which assumptions of classical transport theory are invalid in fully ionized plasma?

- 1.) Collisions cannot be considered as binary interactions due to the long ranging Coulomb force.
- => For weakly non-ideal (fusion) plasma this problem was treated by the Lenard-Balescu equation, which takes the many-particle character of Coulomb collisions (Debye-Shielding) into account.
- 2.) Influence of external magnetic fields on the free motion of particles in fusion devices (toroidal magnetic field and space plasmas.
- => Gyration, Drifts etc. which we studies in test-particle approach. But these effects have large influence on computing transport coefficients.



Tokamak

A Tokamak contains strong magnetic fields in a toroidal geometry in order to trap plasma for nuclear fusion.

But as we just learned: transport coefficients have been underestimated by orders of magnetide
$\Rightarrow$ Plasma transported away and not trapped long enough for fusion

Neo-classical transport theory

- Strong and inhomogenous magnetic fields occur both in tokamaks (fusion machines) and space plasmas.
- Effect of these strong magnetic fields has been first investigated for toroidal magnetic fields. This research was clearly dedicated for fusion plasmas.
- Combining the effects of strong magnetic fields with methods of classical plasma transport lead to significantly higher transport coefficients.
- The new approach was dubbed **neo-classical transport theory**. (Galeev&Sgdeec 1968, Hinton & Hazeltine 1976, Balescu 1988)

Neo-classical transport theory

- Neo-classic theory was an important step forward in fusion theory (never become that popular for space plasmas) and showed reasonable agreement with some experiments.
- But: Still not satisfactory to explain the often observed leaks in tokamaks.
- Problem: A real plasma is never in a quiescent state (like the static and stationary equilibria we calculated) but collective nature of plasma dominates it's behaviour.
- At high temperatures (both in fusion and space plasmas) particle collisions are less important than self-organized effects like waves, vortices, modes etc.

Collective effects

- Such self-organized coherent structures are more efficients to tranport plasma and energy than interaction of individual particles. (For space-plasma physicists it‘s interesting to study these dynamics, but fusion-people loose the plasma in their machines due to these effects => no fusion.)
- Remember the many waves we studied in space plasmas (and we studied only a small fraction of all known plasma waves and in simplified geometries)
- Some waves become unstable (growing amplitudes) and we studied instabilities mainly with linear theory.
- For large amplitudes nonlinear effects become important.

Complexity and turbulence

- For low aplitude waves linear superposition can be used to study the combined effects.
- As amplitudes grow, nonlinear interactions become very important => One gets a large number of new modes, saturation of amplitudes.
- => The plasma becomes more and more complex and it‘s behavious unpredictable.
- => Turbulent or chaotic motion.
- In chaos small differences in location and velocity can influence significantly the evolution (like the butterfly-effect in weather prediction).

Non-equilibrium steady state

- Under quiet condition the random collisions in a gas lead to a steady state thermal equilibrium.
- For the Boltzmann-equation (Vlasov-equation with Boltzmann collision term) it has been proofed that the final stationary state solution is the Maxwell-Boltzmann distribution for the distribution functions.
- Turbulent, collective interaction (e.g. of particles and waves) will lead to a non-equilibrium steady state.
- Turbulent dissipation becomes very important and does of course influence (or even dominate!) the transport coefficients (additional to classical and neo-classical effects)
- => **Anomalous Transport Theory**

Anomalous vs. strange transport

- In anomalous transport theory the transport coefficients are computed from turbulent/chaotic collective particle motions.
- In many cases the relations between macroscopic plasma variables remain simple, e.g. a linear dependence of fluxes and forces.
- One can still use fluid theory, but with transport coefficients computed by turbulent theory.
  => Macroscopic laws are the same as in classical theory
- In some cases, however, turbulence can lead to strange results, e.g. the transport coefficients are either zero or infinity => **strange transport**.
  (maybe called strange because we not understand it)

Anomalous transport: Quasi linear theory

- Quasi-linear theory, valid for weak turbulence.
- Particle distribution function and fields can be split into a slowly evolving part (f_{s0}, $\mathbf{E}_0$, $\mathbf{B}_0$) and small fluctuations (δf_s, $\delta\mathbf{E}$ and $\delta\mathbf{B}$)
- We get the Vlasov equation for the slowly evolving part:

$$\frac{\partial f_{s0}}{\partial t} + \mathbf{v}\cdot\nabla f_{s0} + \frac{q_s}{m_s}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_{s0}}{\partial \mathbf{v}} = -\frac{q_s}{m_s}\left\langle(\delta\mathbf{E}+\mathbf{v}\times\delta\mathbf{B})\cdot\frac{\partial \delta f_s}{\partial \mathbf{v}}\right\rangle$$

- As we calculated in the excercises, the entropy remains constant in Vlasov systems. If one defines, however, an entropy only for the slowly evolving part, this (new defined) entropy can increase and be used to measure the disorder of a system

Numerical simulations

- In many cases we cannot apply quasi-linear theory or other analytic methods and use numerical simulations.
- Are **simulations** theory or experiments?
- For experimental physicists (or observers in space physics) often everything not related to experiments or observations is theory.
- But: Often not true and we speak of so called: **numerical experiments**. Simulations produces often such huge amount of data, that methods similar as for observational data are used to analyze them.
- If we able to correctly simulate a physical system, this does not necessarily mean that we understand all physics. (But it helps, as we can switch on/off physical terms.)

How to measure complexity – Gauss' linking number

For two interlinked curves (Berger, 1999a):

$$L_{12} := -\frac{1}{4\pi}\int_{l_1}\int_{l_2}(d\boldsymbol{l}_1 \times d\boldsymbol{l}_2)\cdot\frac{\boldsymbol{r}}{r^3} \qquad (1)$$

Sum over every pair of field lines within a volume = magnetic helicity.

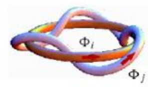
If there are N tubes with flux ϕ_i, $i = 1, \ldots, N$ in a closed volume:

$$H = \sum_{i=1}^{N}\sum_{j=1}^{N} L_{ij}\ \phi_i\ \phi_j = \qquad (2)$$

$N(N-1)$ terms where $i \neq j$: *mutual helicity* between tubes i and j: $2\ L_{ij}\ \phi_i\ \phi_j$ $\quad + \quad$ N terms where $i = j$: *self helicity* of tubes i and j: $L_{ii}\ \phi_i^2$

How to measure complexity – simple examples

$L_{ii} = L_{jj} = 0,\ L_{ij} = -1$
$H = 2\ L_{ij}\ \phi_i\ \phi_j = -2\ \phi_i\ \phi_j$



$L_{ii} = L_{jj} = 0,\ L_{ij} = -3$
$H = 2\ L_{ij}\ \phi_i\ \phi_j = -6\ \phi_i\ \phi_j$

$L_{ii} = 5,\ L_{ij} = 0$
$H = L_{ii}\ \phi_i^2 = 5\ \phi_i^2$

From linkage to magnetic helicity

Remember : $\quad L_{12} := -\frac{1}{4\pi}\int_{l_1}\int_{l_2}(d\boldsymbol{l}_1 \times d\boldsymbol{l}_2)\cdot\frac{\boldsymbol{r}}{r^3} \qquad H = \sum_{i=1}^{N}\sum_{j=1}^{N} L_{ij}\ \phi_i\ \phi_j$

With $\phi_i \to 0$ for $N \to \infty$ this leads to (M. Berger, 1999a):

$$H = -\frac{1}{4\pi}\int\int \boldsymbol{B}(\boldsymbol{x})\cdot\frac{\boldsymbol{r}}{r^3}\times\boldsymbol{B}(\boldsymbol{x}')\,d^3x'\,d^3x$$

$$= \int \boldsymbol{A}(\boldsymbol{x}')\cdot\boldsymbol{B}(\boldsymbol{x})\,d^3x, \qquad (3)$$

$$\boldsymbol{A} = -\frac{1}{4\pi}\int\frac{\boldsymbol{r}}{r^3}\times\boldsymbol{B}(\boldsymbol{x}')\,d^3x' \qquad (4)$$

Magnetic helicity – classical definition

$$H := \int_V \boldsymbol{A}\cdot\boldsymbol{B}\,d^3x \qquad (5)$$

If one wants to use the vector potential $\boldsymbol{A}$, one must ensure gauge invariance (M. Berger, 1999b):

(5) is only valid if:

- V is bounded by a magnetic surface S, i. e. $\boldsymbol{B}\cdot\boldsymbol{n}|_S = 0$
- V is simply connected, i. e. no holes

Inside such surfaces a gauge transform $\boldsymbol{A} \to \boldsymbol{A} + \nabla\psi$ yields

$$\delta H = \int\nabla\psi\cdot\boldsymbol{B}\,d^3x = \int\nabla\cdot(\psi\boldsymbol{B})\,d^3x = \oint\psi\boldsymbol{B}\cdot\boldsymbol{n}|_S\,dS = 0 \qquad (6)$$

- Solar corona:
  - $\boldsymbol{B}\cdot\boldsymbol{n}|_S \neq 0$, i. e. flux is passing through the photospheric boundary
  - classical helicity integral cannot be used

Relative magnetic helicity:

- decomposition of magnetic field in $\boldsymbol{B}_{cl}$ and $\boldsymbol{P}$
- $H_{rel} = \int_V(\boldsymbol{A} + \boldsymbol{A}_P)\cdot(\boldsymbol{B} - \boldsymbol{P})\,d^3x$
- independent of the closure of the field outside the coronal volume

Magnetic Helicity

- Magnetic helicity is a measure on the complexity of magnetic fields.
- Helicity is conserved for ideal MHD-processes and also for some resistive processes like 2D reconnection.
- For 3D reconnection, the helicity can change, but is dissipated much slower as the energy.
- Approximate helicity conservation constrains (additional to energy conservation) which physical processes are possible.
- E.g., A nonlinear force-free magnetic field cannot relax to a potential-field (which would have zero helicity), but only to a linear force-free field with same helicity. (Or the helicity must be transported away, e.g. by a coronal mass ejection).

Fluid vs. Vlasov approach

- In Vlasov theory we study distribution functions f(x,v,t) in 6D-phase space.
- In Fluid equations we deal with macroscopic plasma variables like density n(x,t), flow velocity, V(x,t), plasma pressure p(x,t) in the 3D configuration space.
- What is gained and lost by these approaches?
- The fluid equations are simpler and easier to solve.
- The Vlasov approach is more complete and includes the microphysics.

Fluid vs. Vlasov approach
What is gained by fluid approach?

- It is easy to extend the fluid equations to include collisions, e.g. momentum transfer between ions and electrons or also the inclusion of neural gas atoms. (Computing a collision term in kinetic theory is much more complicated, in particular for anomalous transport)
- One can simplify the fluid equations further by reasonable (for studied system) assumption, e.g. using a scalar pressure instead of tensor or neglection the pressure for low-Beta force-free fields.

Fluid vs. Vlasov approach
What is lost in the fluid approach?

- Fluid equations are not closed (momentum hierarchy) => We have more variables than equations.
- Often it is not easy, however, to make assumptions for pressure, equation of state or energy equation.
- If we can solve f(x,v,t) from Vlasov theory however, computing the pressure tensor and higher moments is just a simple integration => One can deduce an equation of state from Vlasov theory.
- Microphysics effects are lost in fluid theory, in particular all effects in velocity space like Landau damping and kinetic instabilities.

What is missing in Vlasov theory?

- Vlasov equation is basically a fluid flow in 6D-phase-space
- Not included in Vlasov theory are all effects, where the discreteness of the plasma (that plasma is a collection of individual particles) is important:
- What is the level of electromagnetic fluctuations in a stable plasma?
- What is the emission rate and spectrum of radiation from a plasma?
- What is the cross section for scattering of radiation from a plasma?
- => Need higher order kinetic equations.