Space Plasma Physics

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Physical Processes

8. Plasma Waves, instabilities and shocks
9. Magnetic Reconnection

Applications

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11. Solar activity
12. Transport Processes in Plasmas

Electromagnetic wave in a plasma

Electromagnetic wave $\exp(i\omega t - \gamma x)$

With $\gamma = i\frac{\omega}{c}\sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}$ $\qquad w_{pe} = \sqrt{\frac{n_e e^2}{m_e \epsilon_0}} \propto \sqrt{n_e}$

γ becomes real for waves with a frequency $\omega < \omega_{pe}$

Electromagnetic waves with a frequency below the plasma frequency cannot travel into the plasma. (they become reflected) => Cutoff-frequeny

Exercise: How can this property be used to measure the (electron) density of a plasma with EM-waves?

Ideal MHD equations

Plasma equilibria can easily be perturbed and small-amplitude waves and fluctuations can be excited.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0$$

$$\frac{\partial (nm\mathbf{v})}{\partial t} + \nabla \cdot (nm\mathbf{v}\mathbf{v}) = -\nabla \cdot \left(\mathrm{P} + \frac{B^2}{2\mu_0}\mathrm{I}\right) + \frac{1}{\mu_0}\nabla \cdot (\mathbf{B}\mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$\nabla \cdot \mathbf{B} = 0$$

Energy equation omitted, because not needed here.

MHD equilibrium and fluctuations

We assume **stationary** ideal **homogeneous** conditions as the intial state of the single-fluid plasma, with vanishing average electric and velocity fields, overal **pressure equilibrium** and no magnetic stresses. These assumptions yield:

$$\begin{aligned} \mathbf{v}_0 &= 0 \\ \mathbf{E}_0 &= 0 \\ \nabla\left(p_0 + B_0^2/2\mu_0\right) &= 0 \\ (\mathbf{B}_0 \cdot \nabla)\,\mathbf{B}_0 &= 0 \end{aligned}$$

These fields are decomposed as sums of their background initial values and space- and time-dependent **fluctuations** as follows:

$$\begin{aligned} n &= n_0 + \delta n \\ \mathbf{v} &= \delta\mathbf{v} \\ \mathbf{E} &= \delta\mathbf{E} \\ \mathbf{B} &= \mathbf{B}_0 + \delta\mathbf{B} \end{aligned}$$

Linear perturbation theory

Because the MHD equations are nonlinear (advection term and pressure/stress tensor), the fluctuations must be small.
-> Arrive at a uniform set of linear equations, giving the dispersion relation for the eigenmodes of the plasma.
-> Then all variables can be expressed by one, say the magnetic field.

Usually, in space plasma the background magnetic field is sufficiently strong (e.g., a planetary dipole field), so that one can assume the fluctuation obeys: $|\delta\mathbf{B}| \ll B_0$

In the uniform plasma with straight field lines, the field provides the only **symmetry axis** which may be chosen as z-axis of the coordinate system such that: $\mathbf{B}_0 = B_0 \hat{\mathbf{e}}_{||}$.

Linearized MHD equations

Linarization of the MHD equations leads to three equations for the three fluctuations, δn, $\delta\mathbf{v}$, and $\delta\mathbf{B}$:

$$\frac{\partial \delta n}{\partial t} + n_0 \nabla \cdot \delta\mathbf{v} = 0$$

$$m_i n_0 \frac{\partial \delta\mathbf{v}}{\partial t} = -\nabla\left(\delta p + \frac{1}{\mu_0}\mathbf{B}_0 \cdot \delta\mathbf{B}\right) + \frac{1}{\mu_0}(\mathbf{B}_0 \cdot \nabla)\,\delta\mathbf{B}$$

$$\frac{\partial \delta\mathbf{B}}{\partial t} = (\mathbf{B}_0 \cdot \nabla)\,\delta\mathbf{B} - \mathbf{B}_0(\nabla \cdot \delta\mathbf{v})$$

Using the adiabatic pressure law, and the derived sound speed, $c_s^2 = p_0/(m_i n_0)$, leads to an equation for δp and gives:

$$\frac{\partial \delta p}{\partial t} = m_i c_s^2 \frac{\partial \delta n}{\partial t} = -m_i n_0 c_s^2 \nabla \cdot \delta\mathbf{v}$$

Linearized MHD equations II

Inserting the continuity and pressure equations, and using the Alfvén velocity, $\mathbf{v}_A = \mathbf{B}_0/(\mu_0 n m_i)^{1/2}$, two coupled vector equations result:

$$\frac{\partial \delta \mathbf{v}}{\partial t} = v_A^2 \nabla_{\|} \left(\frac{\delta \mathbf{B}_{\perp}}{B_0}\right) - \nabla\left(\frac{\delta p}{m_i n_0}\right)$$
$$\frac{\partial}{\partial t}\left(\frac{\delta \mathbf{B}}{B_0}\right) = \nabla_{\|}\delta \mathbf{v}_{\perp} - \hat{\mathbf{e}}_{\|}\left(\nabla_{\perp}\cdot\delta\mathbf{v}_{\perp}\right)$$

Time differentiation of the first and insertion of the second equation yields a second-order wave equation which can be solved by **Fourier transformation**.

$$\frac{\partial^2 \delta \mathbf{v}}{\partial t^2} = c_{ms}^2 \nabla\left(\nabla\cdot\delta\mathbf{v}\right) + v_A^2\left(\nabla_{\|}^2 \delta\mathbf{v} - \nabla\nabla_{\|}\delta v_{\|} - \hat{\mathbf{e}}_{\|}\nabla_{\|}\nabla\cdot\delta\mathbf{v}\right)$$

Dispersion relation

The ansatz of **travelling plane waves**,

$$\delta\mathbf{v} = \delta\mathbf{v}_0 \exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$$

with arbitrary constant amplitude, $\delta\mathbf{v}_0$, leads to the

$$\left[\left(\omega^2 - k_{\|}^2 v_A^2\right)\mathbf{I} - c_{ms}^2\mathbf{k}\mathbf{k} + \left(\mathbf{k}\hat{\mathbf{e}}_{\|} + \hat{\mathbf{e}}_{\|}\mathbf{k}\right)k_{\|}v_A^2\right]\cdot\delta\mathbf{v}_0 = 0$$

To obtain a nontrivial solution the determinant must vanish, which means

$$\begin{bmatrix} \omega^2 - v_A^2 k_{\|}^2 - c_{ms}^2 k_{\perp}^2 & 0 & -c_s^2 k_{\|} k_{\perp} \\ 0 & \omega^2 - v_A^2 k_{\|}^2 & 0 \\ -c_s^2 k_{\|} k_{\perp} & 0 & \omega^2 - c_s^2 k_{\|}^2 \end{bmatrix} \begin{bmatrix} \delta v_{0x} \\ \delta v_{0y} \\ \delta v_{0\|} \end{bmatrix} = 0$$

Here the **magnetosonic speed** is given by $c_{ms}^2 = c_s^2 + v_A^2$. The wave vector component perpendicular to the field is oriented along the x-axis, $\mathbf{k} = k_{\|}\hat{\mathbf{e}}_z + k_{\perp}\hat{\mathbf{e}}_x$.

Alfvén waves

Inspection of the determinant shows that the fluctuation in the y-direction decouples from the other two components and has the linear dispersion

$$\omega_A = \pm k_{\|} v_A$$

This **transverse wave** travels parallel to the field. It is called **shear Alfvén wave**. It has no density fluctuation and a constant group velocity, $\mathbf{v}_{gr,A} = \mathbf{v}_A$, which is always oriented along the background field, along which the wave energy is transported.

The transverse velocity and **magnetic field components are (anti)-correlated** according to: $\delta v_y/v_A = \pm\, \delta B_y/B_0$, for **parallel (anti-parallel) wave propagation**. The wave electric field points in the x-direction: $\delta E_x = \delta B_y/v_A$

Magnetosonic waves

The remaing four matrix elements couple the fluctuation components, $\delta v_{\|}$ and $\delta v_{\perp}$. The corresponding determinant reads:

$$\omega^4 - \omega^2 c_{ms}^2 k^2 + c_s^2 v_A^2 k^2 k_{\|}^2 = 0$$

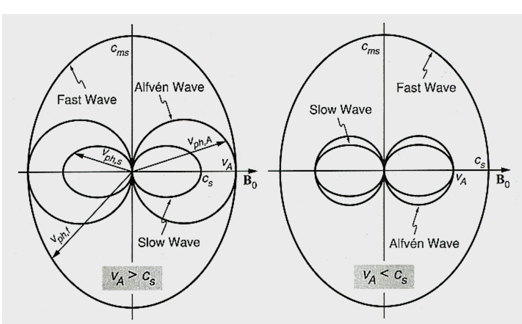
This bi-quadratic equation has the roots:

$$\omega_{ms}^2 = \frac{k^2}{2}\left\{c_{ms}^2 \pm \left[\left(v_A^2 - c_s^2\right)^2 + 4 v_A^2 c_s^2 \frac{k_{\perp}^2}{k^2}\right]^{1/2}\right\}$$

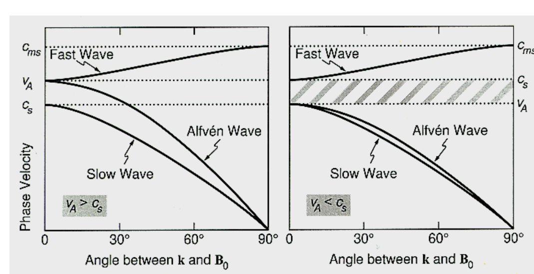
which are the phase velocities of the compressive **fast and slow magnetosonic waves**. They depend on the propagation angle θ, with $k_{\perp}^2/k^2 = \sin^2\theta$. For $\theta = 90^0$ we have: $\omega = k c_{ms}$, and $\theta = 0^0$:

$$\omega^2 = \frac{1}{2}k^2\left[c_s^2 + v_A^2 \pm \left(c_s^2 - v_A^2\right)\right]$$

Phase-velocity polar diagram of MHD waves



Dependence of phase velocity on propagation angle



Magnetosonic wave dynamics

In order to understand what happens physically with the dynamic variables, δv_x, δB_x, $\delta B_{||}$, $\delta v_{||}$, δp, and δn, inspect again the equation of motion written in components:

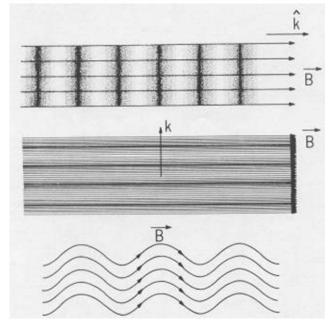
$$\omega \delta \mathbf{v} = \frac{\mathbf{k}}{m_i n_0}\left(\delta p + \frac{1}{\mu_0}\mathbf{B}_0 \cdot \delta \mathbf{B}\right) - \frac{\mathbf{k}\cdot\mathbf{B}_0}{\mu_0 m_i n_0}\delta \mathbf{B}$$

Parallel direction: $$\omega v_{||} = \frac{k_{||}\delta p}{m_i n_0}$$ Parallel pressure variations cause parallel flow.

Oblique direction: $$\omega\left(k_{||}v_{||} + k_{\perp} v_x\right) = \frac{k^2 \delta p_{tot}}{m_i n_0}$$

Total pressure variations ($p_{tot}=p+B^2/2\mu_0$) accelerate (or decelerate) flow, for in-phase (or out-of-phase) variations of δp and δB, leading to the **fast** and **slow mode waves.**

Magnetohydrodynamic waves



- Magnetosonic waves

compressible

- parallel slow and fast
- perpendicular fast

$c_{ms} = (c_s^2+v_A^2)^{1/2}$

- Alfvén wave

incompressible

parallel and oblique

$v_A = B/(4\pi\rho)^{1/2}$

Discontinuities and shocks

Changes occur perpendicular to the discontinuity, parallel the plasma is uniform. The normal vector, **n**, to the surface *S*(**x**) is defined as:

$$\mathbf{n} = -\frac{\nabla S}{|\nabla S|}$$

Any closed line integral (along a rectangular box tangential to the surface and crossing *S* from medium 1 to 2 and back) of a quantity *X* reduces to

$$\oint_S \frac{dX}{dn}dn = 2\int_1^2 \frac{dX}{dn}dn = 2\left(X_2 - X_1\right) = 2[X]$$

Since an integral over a conservation law vanishes, the gradient operation can be replaced by

$$\begin{aligned} \nabla X &\rightarrow \mathbf{n}[X] \\ \nabla\cdot\mathbf{X} &\rightarrow \mathbf{n}\cdot[\mathbf{X}] \\ \nabla\times\mathbf{X} &\rightarrow \mathbf{n}\times[\mathbf{X}] \end{aligned}$$

Transform to a frame moving with the discontinuity at local speed, **U**. Because of ***Galilean invariance***, the time derivative becomes:

$$\partial/\partial t = -\mathbf{U}\cdot\nabla = -U\cdot\mathbf{n}(\partial/\partial n)$$

Discontinuities and shocks

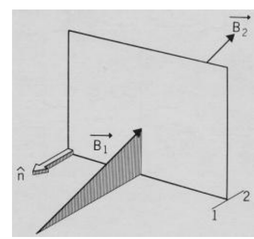
Continuity of the mass flux and magnetic flux:

$B_n = B_{1n} = B_{2n}$

$G_n = \rho_1(V_{1n} - U) = \rho_2(V_{2n} - U)$

U is the speed of surface in the normal direction; **B** magnetic field vector; **V** the flow velocity. ***Mach number***, M = V/C. Here C is the wave phase speed.

Shock:	$G \neq 0$
Discontinuity:	$G = 0$



Contact discontinuity (CD)

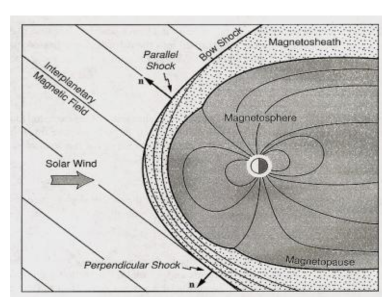
B does not change across the surface of the CD, but $\rho_1 \neq \rho_2$ and $T_1 \neq T_2$.

Rankine-Hugoniot conditions

In the ***comoving frame*** (**v**' = **v** - **U**) the discontinuity (D) is stationary so that the time derivative can be dropped. We ***skip the prime*** and consider the situation in a frame where D is at rest. We assume an isotropic pressure, **P**=p**1**. Conservation laws transform into the ***jump conditions*** across D, reading:

$$\begin{aligned} \mathbf{n}\cdot[n\mathbf{v}] &= 0 \\ \mathbf{n}\cdot[nm\mathbf{v}\mathbf{v}] + \mathbf{n}\left[p + \frac{B^2}{2\mu_0}\right] - \frac{1}{\mu_0}\mathbf{n}\cdot[\mathbf{B}\mathbf{B}] &= 0 \\ [\mathbf{n}\times\mathbf{v}\times\mathbf{B}] &= 0 \\ \mathbf{n}\cdot[\mathbf{B}] &= 0 \end{aligned}$$

Bow shock



The most famous and mostly researched shock is the ***bow shock*** standing in front of the Earth as result of the interaction of the *magnetosphere* with the *supersonic solar wind,* with a high Machnumber, $M_F \approx 8$. Solar wind density and field jump by about a factor of 4 into the *magnetosheath*.

Plasma Instabilities

Because of a multitude of free-energy sources in space plasmas, a very large number of instabilities can develop.

If spatial the involved scale is:

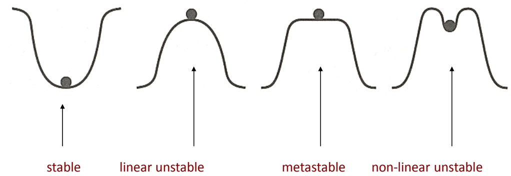
- comparable to macroscopic size (bulk scale of plasma,.....)
  -> ***macroinstability*** (affects plasma globally)
- comparable to microscopic scale (gyroradius, inertial length,...)
  -> ***microinstability*** (affects plasma locally)

Theoretical treatment:

- *macroinstability*, fluid plasma theory
- *microinstability*, kinetic plasma theory

Concept of instability

Generation of instability is the general way of redistributing energy which was accumulated in a non-equilibrium state.



Linear instability

The concept of linear instability arises from the consideration of a linear wave function. Assume any variable (density, magnetic field, etc.) here denoted by *A*, the fluctuation of which is δA, that can be Fourier decomposed as

$$\delta A = \sum_{\mathbf{k}} A_{\mathbf{k}} \exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$$

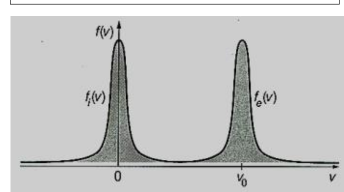
In general the dispersion relation (DR) has complex solutions: $\omega = \omega_r + \gamma$. For real frequency the disturbances are oscillating waves. For complex solutions the sign of γ decides whether the amplitude *A* growths ($\gamma > 0$) or decays ($\gamma < 0$).

$$A_{\mathbf{k}}(t) = A_{\mathbf{k}} \exp\left[\gamma(\omega_r, \mathbf{k})t\right]$$

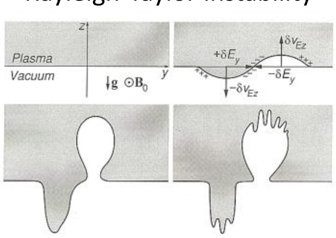
Buneman instability

The electron-ion two-stream instability, ***Buneman instability***, arises from a DR that can be written as (with ions at rest and electrons at speed v_0): => Current disruption

$$\epsilon(\omega, \mathbf{k}) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(\omega - kv_0)^2} = 0$$

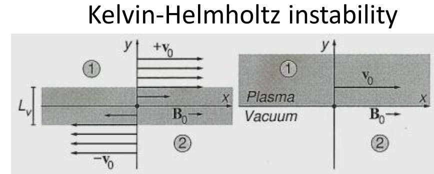


Rayleigh-Taylor instability



Consider a distortion of the boundary so the plasma density makes a sinusoidal excursion. The gravitational field causes an ion drift and current in the negative y direction, $v_{iy} = -m_i g/(eB_0)$, in which electrons do not participate; -> charge separation electric field δE_y evolves. Opposing drifts amplify the original distortions. The bubbles develop similar distortions on even smaller scales.

Kelvin-Helmholtz instability

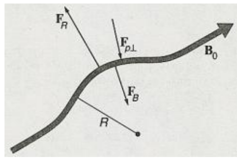


- Shear flow at magnetised plasma boundary may cause ripples on the surface that can grow
- The rigidity of the field provides the dominant restoring force

Consider shear flows (e.g., due to the solar wind) at a boundary, such as between Earth's magnetosheath and magnetopause. Linear perturbation analysis in both regions shows that incompressible waves confined to the interface can be excited.

Firehose instability

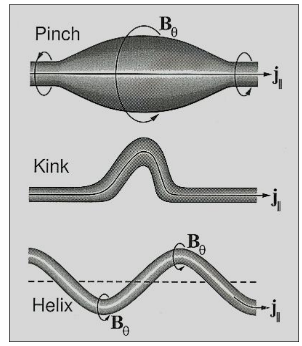
Mechanism of the firehose instability: Whenever the flux tube is slightly bent, the plasma exerts an outward centrifugal force (curvature radius, R), that tends to enhance the initial bending. The gradient force due to magnetic stresses and thermal pressure resists the centrifugal force.



The resulting instability condition for breaking equilibrium is:

$$p_{\|} > p_{\perp} + B_0^2/\mu_0$$

Flux tube instabilities

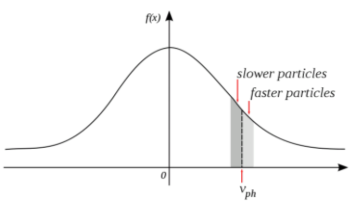


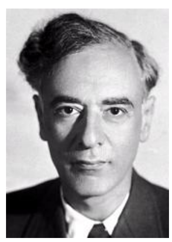
Current disruption

Bending of magnetic field line

Spiral formation of thin flux tube

Landau Damping (Lev Landau 1908-1968, Nobel prize 1962)

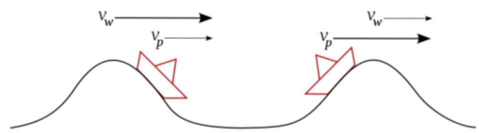




Source: Wikipedia

- Particles slightly slower than the wave gain energy from the wave (and wave looses energy)
- Particles slightly faster than the wave loose energy.
- For a Maxwell distribution there are more slower particles => Wave becomes damped (Landau damping)

Landau-damping



- Landau damping can be compared with a surfer.
- Left: Slow surfer gets catched by the wave and gains energy.
- Right: Fast surfer catches the wave and gives energy to the wave.

Source: Wikipedia

Waves, shocks and instabilities

- Wave occur naturally in fluid and kinetic models, e.g. Alfven waves and magnetosonic waves.
- Shocks are a special case of discontinuieties in fluid model with mass flux across the shock.
- Discontinuities happen in the fluid model, not in nature and for studying the physics of the discontinuity accurately, one has to apply kinetic models.
- Waves with growing amplitudes lead to instabilities, both in kinetic and fluid picture.
- A special instability 'magnetic reconnection' will be treated in the next lecture.