Temperature Correction Schemes

Motivation

Up to now: Radiation transfer in a given atmospheric structure

No coupling between radiation field and temperature included

$\rightarrow$ Including radiative equilibrium into solution of radiative transfer $\rightarrow$ Complete Linearization for model atmospheres (next chapter)

$\rightarrow$ Separate solution via temperature correction

- \+ Quite simple implementation
- \+ Application within an iteration scheme allows completely linear system $\rightarrow$ next chapter
- – No direct coupling
- – Moderate convergence properties

Temperature correction – basic scheme

0. start approximation for $T(\tau) \leftarrow T_0(\tau)$
1. formal solution $J_\nu = \Lambda_\nu S_\nu(T)$
2. correction $T(\tau) \leftarrow T(\tau) + \Delta T(\tau)$
3. convergence?

Several possibilities for step 2 based on radiative equilibrium or flux conservation

Generalization to non-LTE not straightforward

With additional equations towards full model atmospheres:

- Hydrostatic equilibrium
- Statistical equilibrium

LTE

Strict LTE $S_\nu(\tau) = B_\nu(T(\tau))$

Scattering $S_\nu(\tau) = (1-\beta_e)B_\nu(T(\tau)) + \beta_e J_\nu(\tau)$

Simple correction from radiative equilibrium:

$$\int_{\nu=0}^{\infty} \kappa(\tau,\nu)\left(J_\nu(\tau,\nu) - B_\nu(T(\tau),\nu)\right) d\nu \neq 0$$

$$\xrightarrow[\Delta T]{} \int_{\nu=0}^{\infty} \kappa(\tau,\nu)\left(J_\nu(\tau,\nu) - B_\nu\left[T(\tau) + \Delta T(\tau)\right]\right) d\nu = 0$$

$$\Rightarrow \int_{\nu=0}^{\infty} \kappa\left(J_\nu - B_\nu - \Delta T \left.\frac{\partial B_\nu}{\partial T}\right|_{T=T(\tau)}\right) d\nu = 0$$

$$\Rightarrow \Delta T = \int_{\nu=0}^{\infty} \kappa\left(J_\nu - B_\nu\right) d\nu \Big/ \int_{\nu=0}^{\infty} \kappa \left.\frac{\partial B_\nu}{\partial T}\right|_{T=T(\tau)} d\nu = \int_{\nu=0}^{\infty} \kappa\left(J_\nu - B_\nu\right) d\nu \Big/ \int_{\nu=0}^{\infty} \kappa \Delta B_\nu d\nu$$

LTE

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$$\Rightarrow \int_{\nu=0}^{\infty} \kappa\left(J_\nu - B_\nu - \Delta T \left.\frac{\partial B_\nu}{\partial T}\right|_{T=T(\tau)} \right) d\nu = 0$$

$$\Rightarrow \Delta T = \int_{\nu=0}^{\infty} \kappa\left(J_\nu - B_\nu\right) d\nu \Big/ \int_{\nu=0}^{\infty} \kappa \left.\frac{\partial B_\nu}{\partial T}\right|_{T=T(\tau)} d\nu$$

LTE

Problem:

$$\Delta T = \int_{\nu=0}^{\infty} \kappa\left(J_\nu - B_\nu\right) d\nu \Big/ \int_{\nu=0}^{\infty} \kappa \left.\frac{\partial B_\nu}{\partial T}\right|_{T=T(\tau)} d\nu$$

$J_\nu \xrightarrow[\tau\to\infty]{} B_\nu$ independent of the temperature $\Rightarrow \Delta T \to 0$

Gray opacity (κ independent of frequency):

$$\int_{\nu=0}^{\infty} \kappa(\nu)\left(J_\nu - B_\nu\right) d\nu \to \kappa(J - B)$$

$$\to \kappa(J - B - \Delta B) = 0$$

$$\to \kappa(J - B) = \kappa\Delta B$$

$$\xrightarrow[\text{0.Moment equation}]{} \frac{dH}{dt} = \kappa\Delta B$$

deviation from constant flux provides temperature correction

Unsöld-Lucy correction

Unsöld (1955) for gray LTE atmospheres, generalized by Lucy (1964) for non-gray LTE atmospheres

0-th moment: $\frac{dH_\nu}{dt} = \kappa_\nu \left(J_\nu - B_\nu\right)$

$$\int \cdots d\nu \rightarrow \frac{dH}{d\tau} = \frac{\kappa_J}{\kappa_B} J - B \;\;,\;\; \kappa_B B = \int_{\nu=0}^{\infty} \kappa_\nu B_\nu d\nu \;\;,\;\; \kappa_J J = \int_{\nu=0}^{\infty} \kappa_\nu J_\nu d\nu \;\;,\;\; d\tau = \kappa_B dt$$

1st moment: $\frac{dK_\nu}{dt} = \kappa_\nu H_\nu$

$$\int \cdots d\nu \rightarrow \frac{dK}{d\tau} = \frac{\kappa_H}{\kappa_B} H \;\;,\;\; \kappa_H H = \int_{\nu=0}^{\infty} \kappa_\nu H_\nu d\nu$$

now new quantities J', H', K' fulfilling radiative equilibrium (local) and flux conservation (non local)

radiative equilibrium: $\frac{dH'}{d\tau} = \frac{\kappa_J}{\kappa_B} J' - B' = 0$

flux conservation: $\frac{dK'}{d\tau} = \frac{\kappa_H}{\kappa_B} H' = \frac{\kappa_H}{\kappa_B} \frac{\sigma}{4\pi} T_{\text{eff}}^4$

Unsöld-Lucy correction

Now corrections to obtain new quantities:

$$\Delta X = X' - X$$

$$\frac{d\Delta K}{d\tau} = \frac{\kappa_H}{\kappa_B} \Delta H \quad \text{integrate} \quad \rightarrow \Delta K = \Delta K(0) + \int_{\tau'=0}^{\tau} \frac{\kappa_H}{\kappa_B} \Delta H d\tau'$$

$$K = \int_0^\infty K_\nu d\nu = \int_0^\infty f_\nu J_\nu d\nu = fJ \;\;,\;\; H(0) = \int_0^\infty H_\nu(0) d\nu = \int_0^\infty h_\nu J_\nu(0) d\nu = hJ(0)$$

$$\rightarrow \Delta K = \frac{f(0)\Delta H(0)}{h} + \int_{\tau'=0}^{\tau} \frac{\kappa_H}{\kappa_B} \Delta H d\tau' = f \Delta J$$

$$\frac{d\Delta H}{d\tau} = \frac{\kappa_J}{\kappa_B} \Delta J - \Delta B \rightarrow \Delta B = -\frac{d\Delta H}{d\tau} + \frac{\kappa_J}{\kappa_B}\left(\frac{f(0)\Delta H(0)}{fh} + \frac{1}{f}\int_{\tau'=0}^{\tau} \frac{\kappa_H}{\kappa_B} \Delta H d\tau'\right)$$

$$\Delta B = \frac{4\sigma T^3}{\pi} \Delta T = -\frac{dH'}{d\tau} + \frac{dH}{d\tau} + \frac{\kappa_J}{\kappa_B}\left(\frac{f(0)\Delta H(0)}{fh} + \frac{1}{f}\int_{\tau'=0}^{\tau} \frac{\kappa_H}{\kappa_B} \Delta H d\tau'\right)$$

$$\Delta T = \frac{\pi}{4\sigma T^3}\left[\frac{\kappa_J}{\kappa_B} J - B + \frac{\kappa_J}{\kappa_B}\left(\frac{f(0)\Delta H(0)}{fh} + \frac{1}{f}\int_{\tau'=0}^{\tau} \frac{\kappa_H}{\kappa_B} \Delta H d\tau'\right)\right]$$

Unsöld-Lucy correction

$$\Delta T = \frac{\pi}{4\sigma T^3}\left[\frac{\kappa_J}{\kappa_B}J - B + \frac{\kappa_J}{\kappa_B}\left(\frac{f(0)\Delta H(0)}{fh} + \frac{1}{f}\int_{\tau'=0}^{\tau}\frac{\kappa_H}{\kappa_B}\Delta H d\tau'\right)\right]$$

„Radiative equilibrium“ part good at small optical depths but poor at large optical depths $J \rightarrow B$

„Flux conservation“ part good at large optical depths but poor at small optical depths $\frac{dH}{d\tau} \rightarrow 0$

Unsöld-Lucy scheme typically requires damping

Still problems with strong resonance lines, i.e. radiative equilibrium term is dominated by few optically thick frequencies

Unsöld-Lucy correction

Generalization for scattering

0-th moment: $\frac{dH_\nu}{dt} = \kappa_\nu\left(J_\nu - S_\nu^{tot}\right) = \kappa_\nu\left(J_\nu - (1-\beta_e)B_\nu - \beta_e J_\nu\right)$

$$\int \cdots d\nu \rightarrow \frac{dH}{d\tau} = \frac{\kappa_J}{\kappa_B}J - B$$

$$\kappa_B B = (1-\beta_e)\int_{\nu=0}^{\infty}\kappa_\nu B_\nu d\nu \ , \ \kappa_J J = (1-\beta_e)\int_{\nu=0}^{\infty}\kappa_\nu J_\nu d\nu \ , \ d\tau = \kappa_B dt$$

$\vdots$

All the rest is the same

Difficulties for scattering dominated regions: weak coupling between radiation field and temperature

$$\beta_e \rightarrow 1 \Rightarrow \begin{cases} \kappa_B \Delta B \rightarrow 0 \\ \kappa_J \Delta J \rightarrow 0 \\ \frac{d\Delta H}{d\tau} \rightarrow 0 \end{cases}$$

Unsöld-Lucy correction

Generalization to non-LTE (Werner & Dreizler 1998, Dreizler 2003)

0-th moment: $\frac{dH_\nu}{dt} = \kappa_\nu (J_\nu - S_\nu) = \kappa_\nu J_\nu - \tilde{\kappa}_\nu B_\nu - \gamma_\nu J_\nu$

$$\int \cdots d\nu \rightarrow \frac{dH}{d\tau} = \frac{\kappa_J}{\kappa_B} J - B$$

$$\kappa_B B = \int_{\nu=0}^{\infty} \tilde{\kappa}_\nu B_\nu d\nu \ , \ \kappa_J J = \int_{\nu=0}^{\infty} (\kappa_\nu - \gamma_\nu) J_\nu d\nu \ , \ d\tau = \kappa_B dt$$

$\vdots$

All the rest is the same

$\tilde{\kappa}_\nu$ should contain only terms which couple directly to the temperature, i.e. bf and ff transitions

Depth dependent damping (need to play with parameters **c**):

$$\Delta T = \frac{\pi}{4\sigma T^3}\left[c_1 e^{-\tau_0/\tau}\left(\frac{\kappa_J}{\kappa_B} J - B\right) + \frac{\kappa_J}{\kappa_B}\left(c_2 (1-e^{-\tau_0/\tau}) \frac{f(0)\Delta H(0)}{fh} + \frac{c_3(1-e^{-\tau_0/\tau})}{f} \int_{\tau'=0}^{\tau} \frac{\kappa_H}{\kappa_B} \Delta H d\tau' \right)\right]$$

Stellar Atmospheres

This was the contents of our lecture:

Radiation field
Radiation transfer
Emission and absorption
Radiative equilibrium
Hydrostatic equilibrium
Stellar atmosphere models

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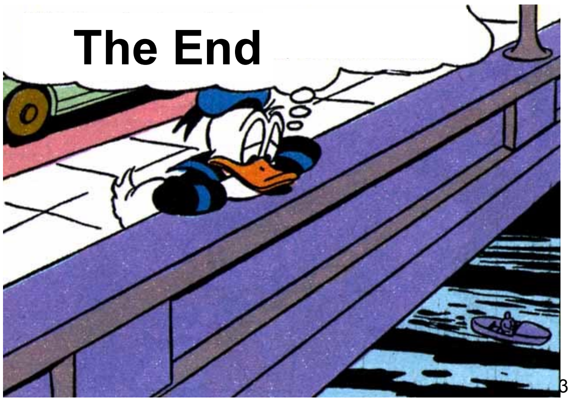

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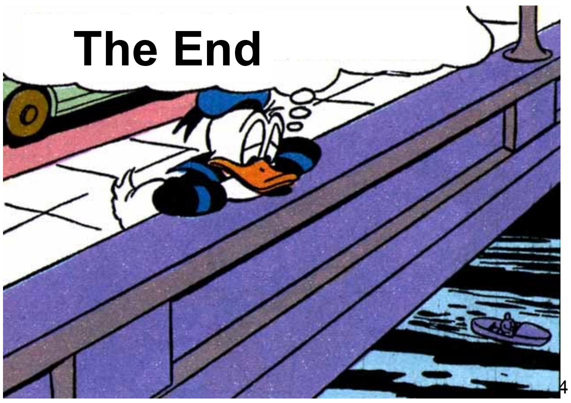
Thank you for listening !



Avrett-Krook method

In case that flux conservation and radiative equilibrium is not fulfilled, Unsöld-Lucy can only change the temperature

Change of other quantities, e.g. opacity, is not accounted for

$\rightarrow$ Avrett & Krook (1963)

strict LTE assumed, generalization straightforward

Current quantities:

$$\mu \frac{dI_\nu^0}{d\tau^0} = \underbrace{\frac{\kappa_\nu^0}{\kappa^0}}_{=\chi_\nu^0} \left(I_\nu^0 - B_\nu^0 (T^0(\tau^0)) \right) \quad \text{with some kind of mean opacity } \kappa^0$$

Does not fulfill flux conservation and radiative equilibrium

New quantities:

$$\mu \frac{dI_\nu}{d\tau} = \underbrace{\frac{\kappa_\nu}{\kappa}}_{=\chi_\nu} \left(I_\nu - B_\nu (T(\tau)) \right) \quad \text{with mean opacity } \kappa$$

Avrett-Krook method

Linear Taylor expansion of the new quantities from old ones:

$$\tau = \tau^0 + \tau^1 \rightarrow \frac{d\tau}{d\tau^0} = 1 + \frac{d\tau^1}{d\tau^0} \quad T = T^0 + T^1 \quad I_\nu = I_\nu^0 + I_\nu^1 \quad H = \int (H_\nu^0 + H_\nu^1) d\nu = \sigma / 4\pi \, T_{\text{eff}}^4$$

$$\chi_\nu = \chi_\nu^0 + \chi_\nu^1 = \chi_\nu^0 + \tau^1 \left. \frac{d\chi_\nu}{d\tau} \right|_0 \quad B_\nu = B_\nu^0 + B_\nu^1 = B_\nu^0 + T^1 \left. \frac{dB_\nu}{dT} \right|_0$$

Radiative transfer equation:

$$\mu \frac{dI_\nu}{d\tau} = \chi_\nu \left(I_\nu - B_\nu(T(\tau)) \right)$$

$$\mu \frac{dI_\nu^0}{d\tau^0} + \mu \frac{dI_\nu^1}{d\tau^0} = \frac{d\tau}{d\tau^0} \left(\chi_\nu^0 + \chi_\nu^1 \right) \left(I_\nu^0 + I_\nu^1 - B_\nu^0 - B_\nu^1 \right)$$

$$\chi_\nu^0 \left(I_\nu^0 - B_\nu^0 \right) + \mu \frac{dI_\nu^1}{d\tau^0} = \left(1 + \frac{d\tau^1}{d\tau^0} \right) \left(\chi_\nu^0 + \chi_\nu^1 \right) \left(I_\nu^0 + I_\nu^1 - B_\nu^0 - B_\nu^1 \right)$$

$$\chi_\nu^0 \left(I_\nu^0 - B_\nu^0 \right) + \mu \frac{dI_\nu^1}{d\tau^0} = \left(\chi_\nu^0 + \chi_\nu^1 + \chi_\nu^0 \frac{d\tau^1}{d\tau^0} \right) \left(I_\nu^0 + I_\nu^1 - B_\nu^0 - B_\nu^1 \right)$$

$$\mu \frac{dI_\nu^1}{d\tau^0} = \chi_\nu^0 \left(I_\nu^1 - B_\nu^1 \right) + \left(\chi_\nu^1 + \chi_\nu^0 \frac{d\tau^1}{d\tau^0} \right) \left(I_\nu^0 - B_\nu^0 \right)$$

Avrett-Krook method

1st moment:

$$\mu \frac{dI_\nu^1}{d\tau^0} = \chi_\nu^0 \left(I_\nu^1 - B_\nu^1\right) + \left(\chi_\nu^1 + \chi_\nu^0 \frac{d\tau^1}{d\tau^0}\right)\left(I_\nu^0 - B_\nu^0\right) \quad \int \cdots \mu d\mu$$

$$\rightarrow \frac{dK_\nu^1}{d\tau^0} = \chi_\nu^0 H_\nu^1 + \left(\chi_\nu^1 + \chi_\nu^0 \frac{d\tau^1}{d\tau^0}\right) H_\nu^0 = \chi_\nu^0 H_\nu^1 + \left(\tau^1 \frac{d\chi_\nu}{d\tau^0} + \chi_\nu^0 \frac{d\tau^1}{d\tau^0}\right) H_\nu^0 := 0$$

$$\rightarrow \frac{d\tau^1}{d\tau^0} H_\nu^0 + \tau^1 \frac{d\chi_\nu}{d\tau^0} \frac{H_\nu^0}{\chi_\nu^0} = -H_\nu^1 \quad \int \cdots d\nu$$

$$\rightarrow \frac{d\tau^1}{d\tau^0} H^0 + \tau^1 \int_0^\infty \frac{d\chi_\nu}{d\tau^0} \frac{H_\nu^0}{\chi_\nu^0} d\nu = H^0 - \frac{\sigma}{4\pi} T_{\mathrm{eff}}^4 \quad \text{, linear DEQ of first order}$$

$$\Rightarrow \tau^1 = \frac{1}{M(\tau^0)} \int_0^{\tau^0} M(x) \left[1 - \frac{\sigma T_{\mathrm{eff}}^4}{4\pi H^0(x)}\right] dx \quad , \quad M(x) = \exp\left(\int_0^x dy \int_0^\infty d\nu \frac{1}{\chi_\nu^0} \frac{d\chi_\nu}{d\tau^0} \frac{H_\nu^0}{H^0}\right)$$

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Avrett-Krook method

Outer boundary:

$$H_\nu^1(0) = h_\nu J_\nu^1(0) \quad \int \cdots d\nu$$

$$H^1(0) = \frac{\sigma}{4\pi} T_{\mathrm{eff}}^4 - H^0(0) = \left(\frac{\sigma T_{\mathrm{eff}}^4}{4\pi H^0(0)} - 1\right) H^0(0) = \left(\frac{\sigma T_{\mathrm{eff}}^4}{4\pi H^0(0)} - 1\right) \int_0^\infty H_\nu^0(0) d\nu$$

$$\rightarrow \int_0^\infty \left(\frac{\sigma T_{\mathrm{eff}}^4}{4\pi H^0(0)} - 1\right) H_\nu^0(0) d\nu = \int_0^\infty h_\nu J_\nu^1(0) d\nu$$

$$\rightarrow J_\nu^1(0) = \left(\frac{\sigma T_{\mathrm{eff}}^4}{4\pi H^0(0)} - 1\right) \frac{H_\nu^0(0)}{h_\nu}$$

$$\frac{dK_\nu^1}{d\tau^0} := 0 \Rightarrow f_\nu J_\nu^1(\tau^0) = \text{const} = f_\nu J_\nu^1(0) = \left(\frac{\sigma T_{\mathrm{eff}}^4}{4\pi H^0(0)} - 1\right) \frac{H_\nu^0(0)}{h_\nu}$$

$$\Rightarrow J_\nu^1(\tau^0) = \left(\frac{\sigma T_{\mathrm{eff}}^4}{4\pi H^0(0)} - 1\right) \frac{H_\nu^0(0)}{f_\nu h_\nu}$$

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Avrett-Krook method

0-th moment:

$$\mu \frac{dI^1_\nu}{d\tau^0} = \chi^0_\nu \left(I^1_\nu - B^1_\nu\right) + \left(\chi^1_\nu + \chi^0_\nu \frac{d\tau^1}{d\tau^0}\right)\left(I^0_\nu - B^0_\nu\right) \qquad \int \cdots d\mu$$

$$\rightarrow \frac{dH^1_\nu}{d\tau^0} = \chi^0_\nu \left(J^1_\nu - B^1_\nu\right) + \left(\chi^1_\nu + \chi^0_\nu \frac{d\tau^1}{d\tau^0}\right)\left(J^0_\nu - B^0_\nu\right)$$

$$\rightarrow \frac{dH^1_\nu}{d\tau^0} = \chi^0_\nu \left(J^1_\nu - T^1 \frac{dB_\nu}{dT^0}\right) + \left(\tau^1 \frac{d\chi_\nu}{d\tau^0} + \chi^0_\nu \frac{d\tau^1}{d\tau^0}\right)\left(J^0_\nu - B^0_\nu\right) \qquad \int \cdots d\nu$$

$$\rightarrow \frac{dH^1}{d\tau^0} = -\frac{dH^0}{d\tau^0} = \int_0^\infty \chi^0_\nu J^1_\nu d\nu - T^1 \int_0^\infty \chi^0_\nu \frac{dB_\nu}{dT^0} d\nu + \tau^1 \int_0^\infty \frac{d\chi_\nu}{d\tau^0}\left(J^0_\nu - B^0_\nu\right) d\nu + \frac{d\tau^1}{d\tau^0}\frac{dH^0}{d\tau^0}$$

$$\Rightarrow T^1 = 1 \Big/ \int_0^\infty \chi^0_\nu \frac{dB_\nu}{dT^0} d\nu$$

$$\left[\tau^1 \int_0^\infty \frac{d\chi_\nu}{d\tau^0}\left(J^0_\nu - B^0_\nu\right) d\nu + \left(1 + \frac{d\tau^1}{d\tau^0}\right)\frac{dH^0}{d\tau^0} + \left(\frac{\sigma T^4_{\mathrm{eff}}}{4\pi H^0(0)} - 1\right)\int_0^\infty \chi^0_\nu \frac{H^0_\nu(0)}{f_\nu h_\nu} d\nu\right]$$

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Radiative equilibrium and Complete Linearization (LTE)

Simultaneous solution of RT and RE radiation transfer:

$$J_\nu(\nu,\tau) - \frac{d^2 J_\nu(\nu,\tau)}{d\tau(\nu)^2} - B_\nu(\nu, T(\tau)) = 0$$

$$\rightarrow f_{ik}(\vec{J}_k, \vec{T}) = 0 \quad \vec{J}_k = \left(J_{1,k}, \cdots, J_{i,k}, \cdots, J_{ND,k}\right) \quad \vec{T} = (T_1, \cdots, T_i, \cdots, T_{ND})$$

$$\vec{J}^0_k, \vec{T}^0 \quad f_{ik}(\vec{J}^0_k, \vec{T}^0) \neq 0 \Rightarrow \text{correction } \delta\vec{J}_k, \delta\vec{T} \rightarrow f_{ik}(\vec{J}^0_k + \delta\vec{J}_k, \vec{T}^0 + \delta\vec{T}) = 0$$

$$\text{Taylor expansion: } f_{ik}(\vec{J}^0_k + \delta\vec{J}_k, T^0_i + \delta T)$$

$$= 0 = f_{ik}(\vec{J}^0_k, \vec{T}^0) + \frac{\partial f_{ik}}{\partial J_{i-1k}}\delta J_{i-1k} + \frac{\partial f_{ik}}{\partial J_{ik}}\delta J_{ik} + \frac{\partial f_{ik}}{\partial J_{i+1k}}\delta J_{i+1k} + \frac{\partial f_{ik}}{\partial T_i}\delta T_i$$

$$\rightarrow T_k \delta\vec{J}_k + U_k \delta\vec{T} = \vec{K}_k$$

$$T_k : \text{tri-diagonal with usual } -A_{ik}, B_{ik}, -C_{ik}$$

$$U_k : \text{ diagonal } \left(U_k\right)_{ii} = -\frac{\partial B_\nu(\nu_k, T_i)}{\partial T_i}$$

$$\left(\vec{K}_k\right)_i = -f_{ik}(\vec{J}^0_k, \vec{T}^0)$$

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Radiative equilibrium and Complete Linearization (LTE)

Simultaneous solution of RT and RE radiative equilibrium:

$$\int_{\nu=0}^{\infty} \kappa(\nu,\tau_i)\left(J_\nu(\nu,\tau_i)-B_\nu(\nu,T_i)\right)d\nu=0$$

$$\rightarrow f_{i,NF+1}\left(J_{i,1},\cdots,J_{i,k},\cdots,J_{i,NF},T_i\right)=\sum_{k=1}^{NF} w_k\left(J_{ik}-B_\nu(\nu_k,T_i)\right)=0$$

$$\text{Taylor expansion: } f_{i,NF+1}\left(\vec{J}_k^0+\delta\vec{J}_k,T_i^0+\delta T\right)_{k=1,NF}$$

$$=0=f_{i,NF+1}(\vec{J}_k^0,\vec{T}^0)+\sum_{k=1}^{NF}\frac{\partial f_{i,NF+1}}{\partial J_{ik}}\delta J_{ik}+\frac{\partial f_{i,NF+1}}{\partial T_i}\delta T_i \rightarrow \sum_{\mathrm{k}=1}^{\mathrm{NF}} W_k\delta\vec{J}_k+D\delta\vec{T}=\vec{L}$$

W_k : diagonal $\left(W_k\right)_{ii}=w_k$

D: diagonal $\left(D\right)_{ii}=-\sum_{k'=1}^{NF} w_{k'}\frac{\partial B_\nu(\nu_{k'},T_i)}{\partial T_i}$

$$\left(\vec{L}\right)_i=-\sum_{k=1}^{NF} w_k\left(J_{ik}^0-B_\nu(\nu_k,T_i^0)\right)$$

Radiative equilibrium and Complete Linearization (LTE)

Together: Rybicki scheme:

$$\begin{pmatrix} T_1 & & & & & U_1 \\ & \ddots & & 0 & & \vdots \\ & & T_k & & & U_k \\ & 0 & & \ddots & & \vdots \\ & & & & T_{NF} & U_{NF} \\ \hline W_1 & & W_k & & W_{NF} & D \end{pmatrix} \begin{bmatrix} \delta\vec{J}_1 \\ \vdots \\ \delta\vec{J}_k \\ \vdots \\ \delta\vec{J}_{NF} \\ \hline \delta\vec{T} \end{bmatrix} = \begin{bmatrix} \vec{K}_1 \\ \vdots \\ \vec{K}_k \\ \vdots \\ \vec{K}_{NF} \\ \hline \vec{L} \end{bmatrix}$$

RE takes the part of the scattering integral

Instead of $\overline{J}$ solve for temperature corrections

Non-linear $\rightarrow$ iteration

During the iteration: new opacities, Eddington factors

Radiative equilibrium and Complete Linearization (NLTE)

Direct generalization at least problematic due to weak coupling of NLTE source function to the temperature

Take into account the change of the population numbers

$\rightarrow$ Add RE to the Complete Linearization scheme (Auer & Mihalas 1969)

Radiative equilibrium in NLTE:

$$\int_{\nu=0}^{\infty} \kappa(\nu,\tau_i)\left(J_\nu(\nu,\tau_i)-S_\nu(\nu,\tau_i)\right)d\nu=0$$

$$\rightarrow f_{i,NF+NL+1}\left(J_{i,1},\cdots,J_{i,NF},n_{i,1},\cdots,n_{i,NL},T_i\right)=\sum_{k=1}^{NF} w_k\left(\kappa(\nu_k,\tau_i)J_{ik}-\eta_\nu(\nu_k,\tau_i)\right)=0$$

Linearization: $f_{i,NF+NL+1}(\vec{\psi}^0+\delta\vec{\psi})=$

$$w_k\left(J_{ik}\frac{\partial\kappa_k}{\partial n_{il}}-\frac{\partial\eta_k}{\partial n_{il}}\right)$$

$$0=f_{i,NF+NL+1}(\vec{\psi}^0)+\sum_{k=1}^{NF}\frac{\partial f_{i,NF+NL+1}}{\partial J_{ik}}\delta J_{ik}+\sum_{l=1}^{NL}\frac{\partial f_{i,NF+NL+1}}{\partial n_{il}}\delta n_{il}+\frac{\partial f_{i,NF+NL+1}}{\partial T_i}\delta T_i$$

Radiative equilibrium and Complete Linearization (NLTE)

Together with RT and SE:

$$f_{i,\alpha}(\psi^0+\delta\psi)= \qquad \alpha=1\cdots NF+NL$$

$$0=f_{i,\alpha}(\psi^0)+\sum_{i=1}^{ND}\left\{\sum_{k=1}^{NF}\frac{\partial f_{i,\alpha}}{\partial J_{i,k}}\delta J_{i,k}+\sum_{l=1}^{NL}\frac{\partial f_{i,\alpha}}{\partial n_{i,l}}\delta n_{i,l}+\frac{\partial f_{i,\alpha}}{\partial T_i}\delta T_i\right\}$$

$$-A_i\delta\vec{\psi}_{i-1}+B_i\delta\vec{\psi}_i-C_i\delta\vec{\psi}_{i+1}=\vec{L}_i$$

$$-\begin{pmatrix} \ddots & 0 & \\ A_{i,k} & & 0 \\ 0 & \ddots & \\ & & \\ 0 & & 0 \\ & & \\ 0 & & 0 \end{pmatrix}\begin{bmatrix}\delta\vec{J}_{i-1}\\ \\ \delta\vec{n}_{i-1}\\ \delta T_{i-1}\end{bmatrix}+\begin{pmatrix} \ddots & 0 & & \square & \square & \square \\ & B_{i,k} & & \square & \square & \square \\ 0 & & \ddots & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \end{pmatrix}\begin{bmatrix}\delta\vec{J}_i\\ \\ \delta\vec{n}_i\\ \delta T_i\end{bmatrix}-\begin{pmatrix} \ddots & 0 & \\ C_{i,k} & & 0 \\ 0 & \ddots & \\ & & \\ 0 & & 0 \\ & & \\ 0 & & 0 \end{pmatrix}\begin{bmatrix}\delta\vec{J}_{i+1}\\ \\ \delta\vec{n}_{i+1}\\ \delta T_{i+1}\end{bmatrix}=-\begin{bmatrix}\square\\ \square\\ \square\\ f_{i,\alpha}(\psi^0)\\ \square\\ \square\\ \square\end{bmatrix}$$

Radiative equilibrium and Complete Linearization (NLTE)

Matrix B_i:

$$
B_i = \left(\begin{array}{ccc|ccc|c}
\ddots & & 0 & \vdots & & & \\
& B_{i,k} & \cdots & -\dfrac{\partial S_{i,k}}{\partial n_{i,l'}} & \cdots & & -\dfrac{\partial S_{i,k}}{\partial T_i} \\
0 & & \ddots & \vdots & & & \\
& \vdots & & \vdots & & & \\
\hline
\cdots & \displaystyle\sum_{m=1}^{NL} \frac{\partial (P_i)_{l,m}}{\partial J_{i,k'}} n_{i,m} & \cdots\ \cdots & (P_i)_{l,l'} & \cdots & & \dfrac{\partial (P_i)_{l,m}}{\partial T_i} T_i \\
& \vdots & & \vdots & & & \\
\hline
& \dfrac{\partial f_{iNF+NL+1}}{\partial J_{i,k'}} & & \dfrac{\partial f_{iNF+NL+1}}{\partial n_{i,l'}} & & & \dfrac{\partial f_{iNF+NL+1}}{\partial T_i}
\end{array}\right)
$$

Columns: 1 … NF | 1 … NL | T

Rows: 1 … NF | 1 … NL | T