Stellar Atmospheres in Non-LTE

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radiation transfer
radiative equilibrium
hydrostatic equation

} solve consistently (*) I_ν or J_ν, T, N

Parameters: $T_{\rm eff}$, $\log g$, y_k

LTE: $S_\nu = B_\nu$ strict LTE

$S_\nu = \rho J_\nu + (1-\rho) B_\nu$ including scattering

Population numbers by Saha-Boltzmann equations

NLTE: $\frac{dn_i}{dt} = 0 \rightarrow \underline{\underline{A}}\,\underline{n} = \underline{b}$ rate equations

Solution Methods

(*) is a non-linear system of equations, we look for the solution vector:

$$\underline{\psi}^d = \left(n_1, \cdots, n_{NL}, N, T, n_e, J_{\nu_1}, \cdots, J_{\nu_{NF}}\right)^d$$

d=depth index

with $\underline{\underline{M}}^d\left(\underline{\psi}\right)\underline{\psi}^d = \underline{c}^d\left(\underline{\psi}\right)$

Solution principle: Newton-Raphson iteration

Solution methods:

1. Complete Linearization (Auer & Mihalas 1969)
2. Multi-frequency/multi-gray (Anderson 1985)
3. ALI method (Werner, Husfeld 1985, 1986)

All methods have in common: linearization and iteration

For that, it is necessary to invert matrices (Jacobi matrix) with rank = number of equations

Numerical limit: matrix inversion limits rank to the order of ~ 100

Complete Linearization

Linearizes all equations

Enabled break-through for first calculation of NLTE models, quite robust method

Depth coupling by radiation transfer

⇒ Feautrier scheme

Disadvantage: Capacity limit quickly reached

e.g. model atom with hydrogen and helium:

20 NLTE levels
80 frequency points
} 100 equations

(5 for each spectral line, 2 for each bound-free edge)

Only rather rudimentary representation of plasma

⇒ Number of equations must be reduced

Anderson‘s method

Does not linearize the transfer equation with respect to all frequency points. First: grouping of frequency points in energy blocks. Then: linearization of these quantities.

Number of blocks determines the dimension of the system of equations.

In some sense related to multi-grid methods.

Very clever method, BUT: requires physical motivation for grouping of frequencies. Must be done manually, quite cumbersome, much experience and physical insight by user necessary. Was essentially used by inventor himself, is not used any more.

ALI method

Accelerated Lambda Iteration

Eliminates the explicit inclusion of the transfer equation into the linearization scheme by using instead an implicit approximate solution for J_ν:

Lambda iteration:

$$J_\nu^{(i)} = \Lambda S_\nu^{(i-1)}\left(\underline{n}^{(i-1)}\right)$$

$$\underline{\underline{A}}\left(J_\nu^{(i)}\right)\underline{n}^{(i)} = \underline{b}$$

i=iteration counter

ALI:

$$J_\nu^{(i)} = \Lambda S_\nu^{(i-1)}\left(\underline{n}^{(i-1)}\right) + \Lambda^* S_\nu^{(i)}\left(\underline{n}^{(i)}\right) - \Lambda^* S_\nu^{(i-1)}\left(\underline{n}^{(i-1)}\right)$$

$$J_\nu^{(i)} = \Lambda^* S_\nu^{(i)}\left(\underline{n}^{(i)}\right) + \Delta J_\nu^{(i-1)}$$

$$\underline{\underline{A}}\left(J_\nu^{(i)}\right)\underline{n}^{(i)} = \underline{b}$$

ALI method

Advantage: number of frequency points no longer appears in dimension of equation system to be linearized (but calculation of derivatives of η_ν, κ_ν w.r.t. source function)

No explicit depth coupling, i.e. local linearized equations for every depth point

Starting solution $\underline{\psi}^d = \left(n_1, \cdots, n_{NL}, N, T, n_e\right)^d$

Calculate correction $\underline{\delta\psi}^d = \left(\delta n_1, \cdots, \delta n_{NL}, \delta N, \delta T, \delta n_e\right)^d$

from linearized equation $\underline{\underline{M}}^d \, \underline{\delta\psi}^d = \underline{c}^d$

$$\underline{\delta\psi}^d = \left(\underline{\underline{M}}^d\right)^{-1} \underline{c}^d$$

Improved solution $\underline{\psi}^d + \underline{\delta\psi}^d \rightarrow \underline{\psi}^d$

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