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Solutions to Exercise Sun-Planet Connections (2009)

Fluxgate Magnetometer

2.1 Measurement principle

Single coil system

We integrate the induction equation along the coil line,

$$\nabla \times \vec{E} = -\partial_t \vec{B}. \tag{1}$$

The lhs yields the voltage at the pickup coil

$$V = \int (\nabla \times \vec{E}) \cdot d\vec{s}, \tag{2}$$

where $d\vec{s}$ is the line element along the coil. The rhs becomes

$$\int (-\partial_t \vec{B}) \cdot d\vec{s} = -N\partial_t \int \vec{B} \cdot d\vec{s} \tag{3}$$

$$= -N\partial_t \int \vec{B} \cdot d\vec{S} \tag{4}$$

$$= -NS\partial_t B_x. \tag{5}$$

Here the Stokes theorem was used and the line integral is replaced by the surface integral. B_x is the magnetic field component along the coil axis, and N and S are the winding number of the coil and the coil area, respectively. The integrated induction equation reads

$$V = -NS\partial_t B_x. \tag{6}$$

Therefore with the single coil system one can determine the magnetic field variation.

Double coil system

The background field can be determined in a double coil system. One coil is used as an excitation and the other is used as a pickup. The excitation coil imposes a sine wave pattern in the magnetic field $H(t)$

$$H(t) = H_0 + he^{i\omega t}, \tag{7}$$

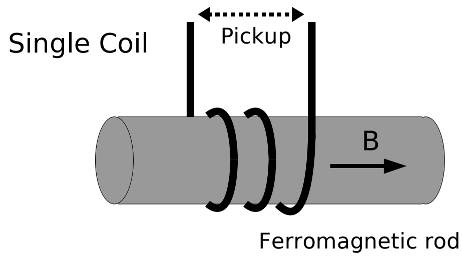

Figure 1: Single coil system.

where the first term on rhs is the background magnetic field (and we aim to measure it), and the second term the excitation part. The excitation is large enough such that the ferromagnetic core is saturated at every half-cycle. The induction field $B(t)$ at the pickup coil is given by the hysterisis curve (B-H curve). For simplicity we approximate the curve as

$$B(t) = H(t) - H^3(t), \tag{8}$$

dropping higher order terms. Using the excitation field $H(t)$ and the B-H curve, the voltage at the pickup coil is given as

$$\begin{aligned} V &= -NS\partial_t B & (9)\\ &= -NS\frac{d}{dt}(H - H^3) & (10)\\ &= -NS[i\omega h(1 - 3H_0)e^{i\omega t} - i6\omega h^2 H_0 e^{i2\omega t} - i3\omega h^3 e^{i3\omega t}]. & (11) \end{aligned}$$

The output voltage therefore contains harmonics of the excitation signal, $n\omega$, and the second harmonics is proportional to the background magnetic field H_0. If we filter the second harmonics only, the output is

$$V = 6NS\omega h^2 H_0 e^{i2\omega t}. \tag{12}$$

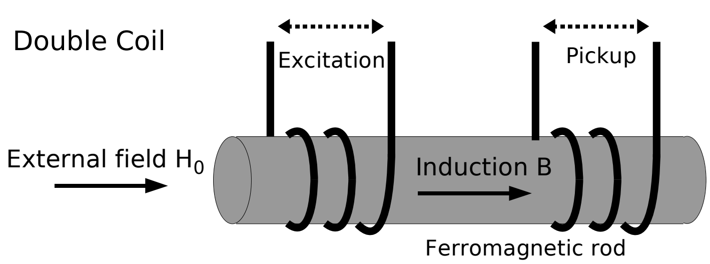

Figure 2: Double coil system.

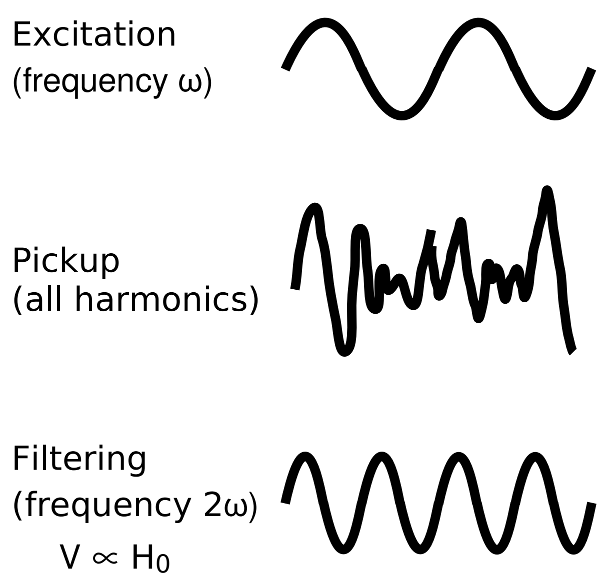

Figure 3: Signal at the excitation, the raw output at the pickup, and the filtered pickup signal.

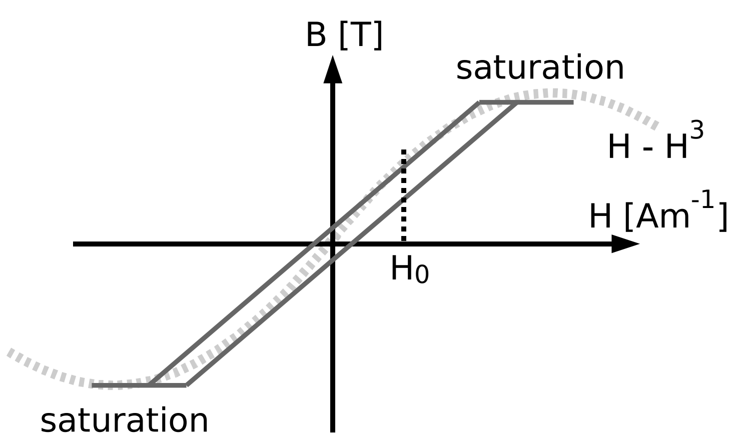

Figure 4: Hysterisis curve of the ferromagnetic rod.

Fluxgate magnetometer

Fluxgate magnetometer is an application of the double coil system. The double rod fluxgate uses two excitation coils in an opposite winding sense, so that all the odd harmonics are canceled out at the pickup. The ringcore fluxgate uses one excitation only, but the ringcore geometry works equivalently as the double rod fluxgate. Excitation frequency is usually in the kHz regime.

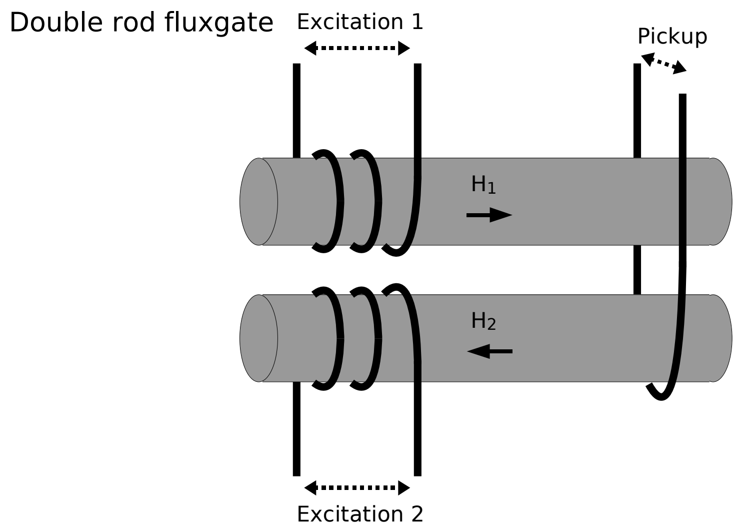

Figure 5: Double rod fluxgate magnetometer.

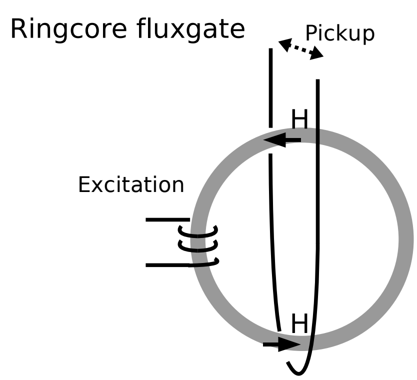

Figure 6: Ringcore fluxgate magnetometer.

2.1 Assembly

The capacitor C is used to form an oscillatory circuit together with the pickup coil L_2. The oscillation frequency is chosen to filter the second harmonics of the excitation frequency $2f$,

$$(2\pi \times 2f)^2 = \frac{1}{L_2 C}. \tag{13}$$

Setting the drive frequency $f = 8\,[\text{kHz}]$ and $L_2 = 13\,[\text{mH}]$, we obtain $C \simeq 8\,[\text{nF}]$.

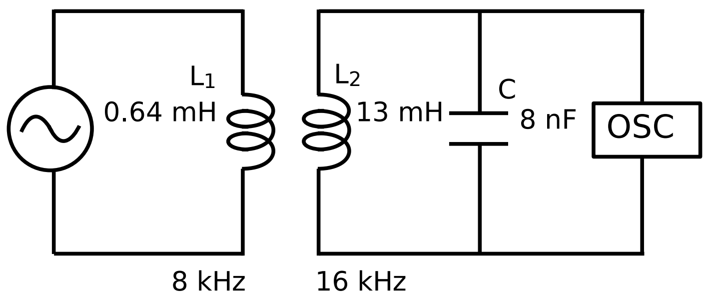

Figure 7: Assembled fluxgate magnetometer.