

Magnetic field in electrically
conductive medium (fluid, gas)

Electric field

$$\vec{E} = \underbrace{-\vec{v} \times \vec{B}}_{\text{convective}} + \underbrace{\frac{1}{\sigma} \vec{j}}_{\text{ohmic}} \quad \dots (1)$$

$$= -\vec{v} \times \vec{B} + \frac{1}{\mu_0 \sigma} \nabla \times \vec{B} \quad \dots (2)$$

(Ampère's law)

Induction eq.

$$\partial_t \vec{B} = -\nabla \times \vec{E} \quad \dots (3)$$

Combine (2) and (3) and $\nabla \cdot \vec{B} = 0$

↘

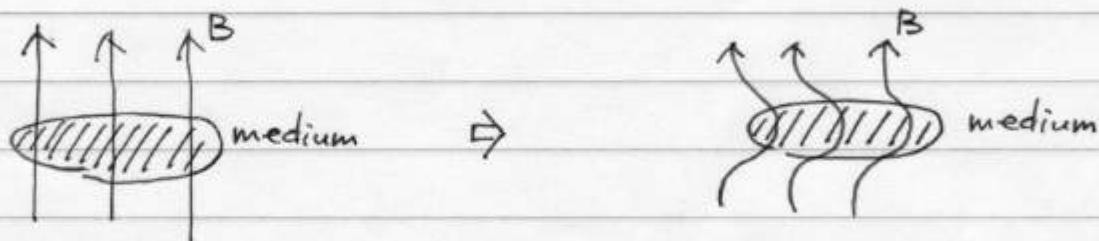
$$\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B} \quad \dots (4)$$

Evolution of
mag. field

(A) frozen-in

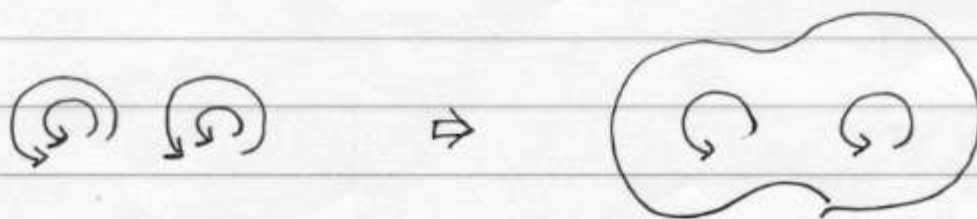
(B) diffusion

(A) Frozen-in magnetic field



Magnetic field moves with flow/medium.

(B) Diffusion



Magnetic field becomes weaker.

Space plasma ... collisionless, conductive medium.
Conductivity σ very large.

↘ Frozen-in magnetic field

Consequences

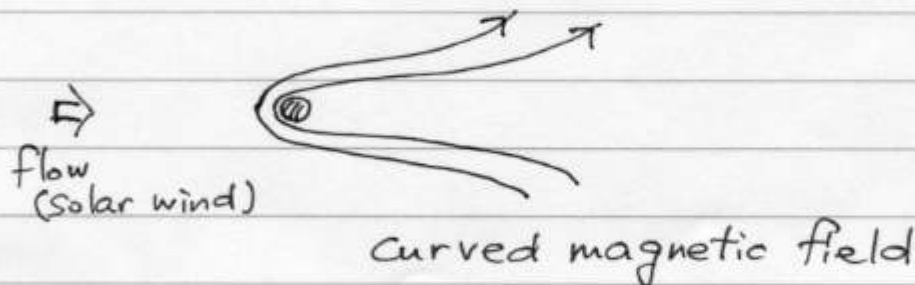
(1) Magnetic field moves with flow
(e.g. solar wind)

(2) Different media cannot be mixed.

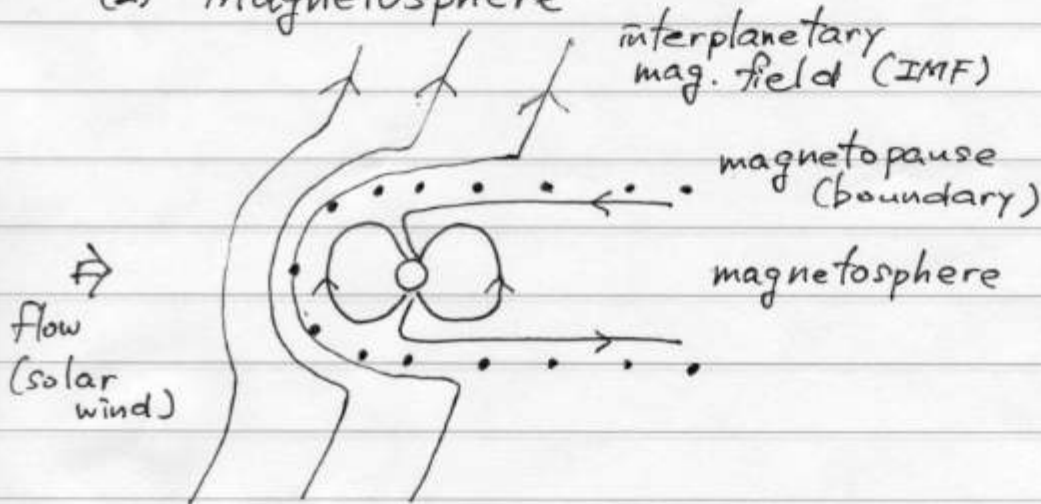
↘ Formation of boundaries
(e.g. magnetopause)

Applications

(1) Comet tail (Alfvén's idea, 1957)



(2) Magnetosphere



How large is magnetosphere?

Pressure balance

$$\frac{1}{2} \rho v^2 = \frac{B^2}{2\mu_0} \quad \dots (5)$$

dynamic pressure	=	magnetic pressure
(kinetic energy density)		(mag. energy density)

solar wind

planet

Dipole magnetic field (for planet)

$$B \propto \frac{1}{r^3}$$

$$\Rightarrow \frac{B}{B_{\text{surf}}} = \left(\frac{R_{\text{surf}}}{R_{\text{mp}}} \right)^3 \quad \dots (6)$$

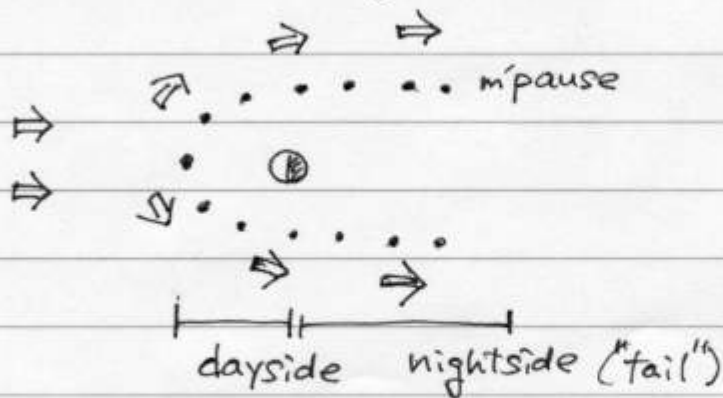
Combine (5) and (6)

$$\Rightarrow \frac{R_{\text{mp}}}{R_{\text{surf}}} = \left(\frac{B_{\text{surf}}}{\mu_0 \rho V^2} \right)^{1/6} \quad \dots (7)$$

Classification of planets

	$R_{\text{mp}} > R_{\text{surf}}$	$R_{\text{mp}} < R_{\text{surf}}$
Solid surface	Mercury	Earth Moon
gas surface (atmosphere)	Earth Jupiter, Saturn Uranus, Neptune	Venus, Mars

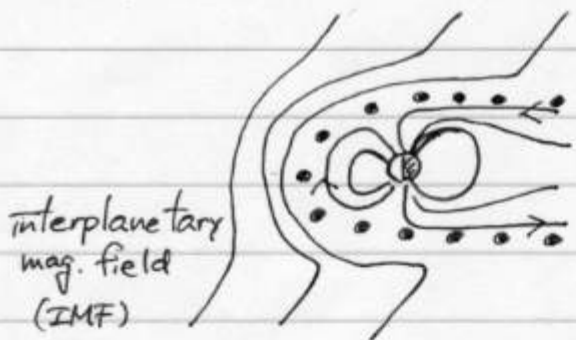
• Flow around magnetosphere



Surface mag. field 30 000 nT

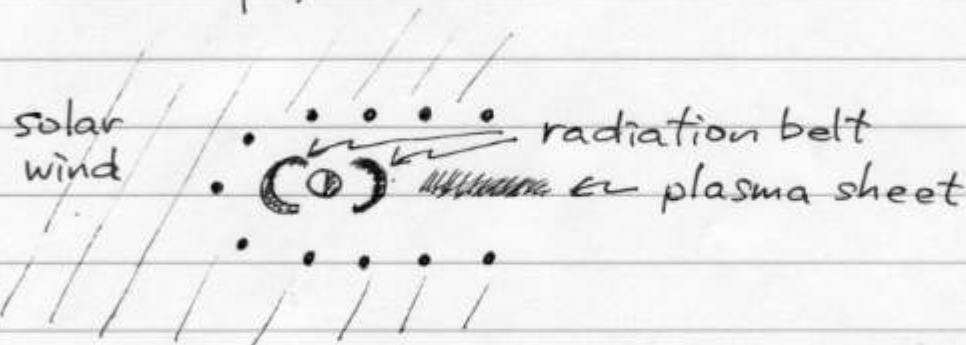
Magnetopause distance ... 10 ~ 11 R_E (dayside)

• Magnetic field



solar wind ... curved mag. field
 dayside magnetosphere ... dipolar field
 nightside ... elongated / stretched field

• Plasma populations

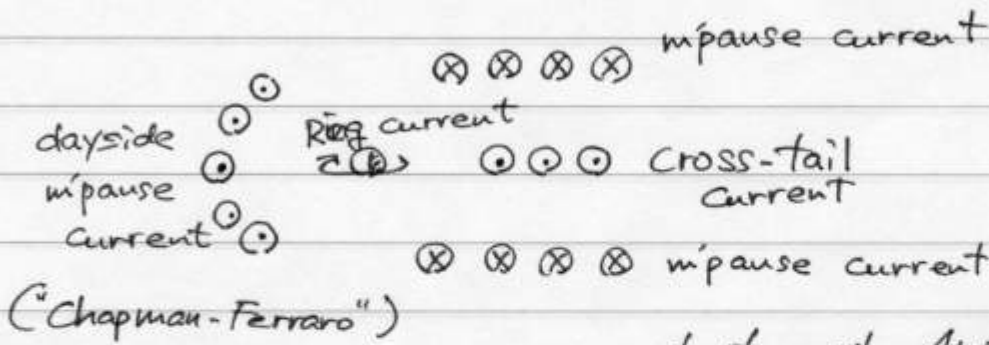


solar wind

radiation belt

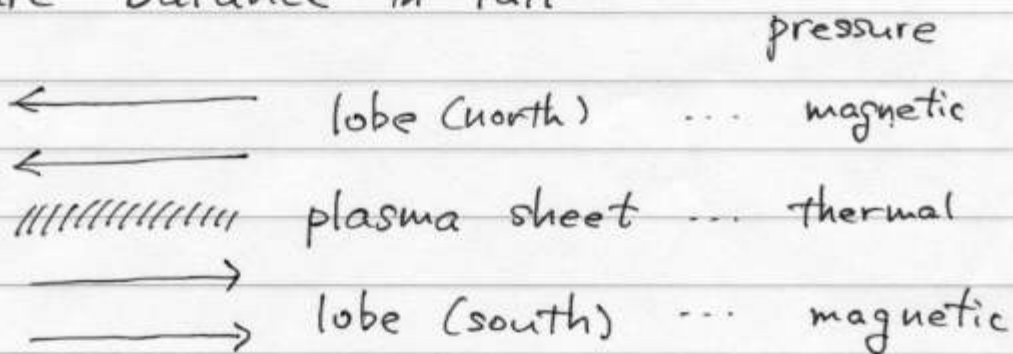
plasma sheet

• Electric current



consistent with Ampère's law
~~discontinuous current~~ $\vec{j} = \frac{1}{\mu_0} \nabla \times \vec{B}$

• Pressure balance in tail

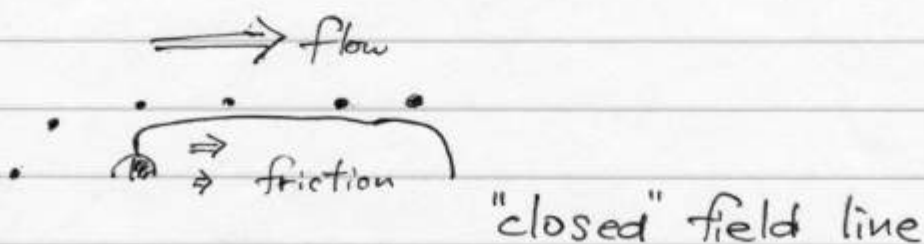


$$P_{\text{plasma sheet}} = P_{\text{magnetic lobe}}$$

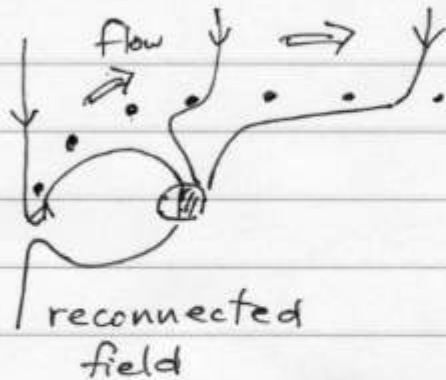
• Why is tail-field stretched?

possibility 1 ... viscosity/friction

(Axford and Hines, 1961)

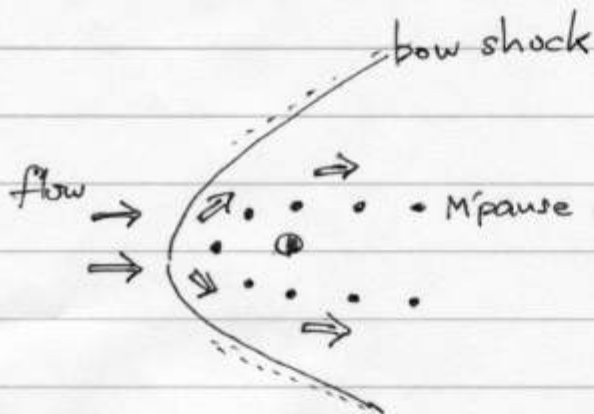


possibility 2 ... reconnection (Dungey 1961)



"open" field line

• bow shock



- standing shock wave
- misphere as obstacle
- flow speed changes from super-magnetosonic to sub-magnetosonic

cf. magnetosonic speed

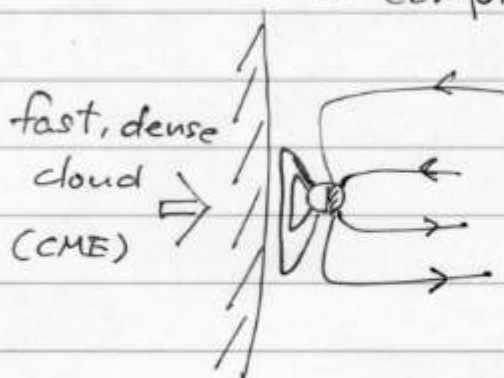
$$V \sim \sqrt{V_A^2 + C_s^2}$$

- plasma is ⁽¹⁾ compressed, ⁽²⁾ heated

- magnetic field also compressed

Geomagnetic storm

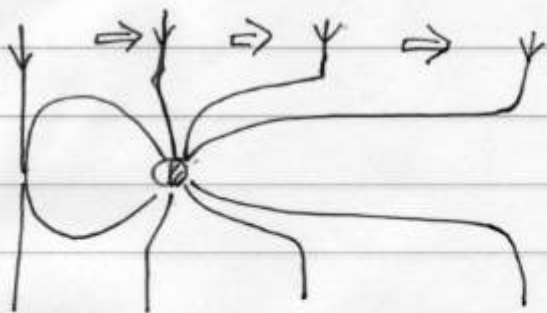
... compression of magnetosphere
(Chapman, 1930)



1. dayside compression ~ minutes
2. nightside compression ~ hours
3. recovery phase (expansion) ~ days

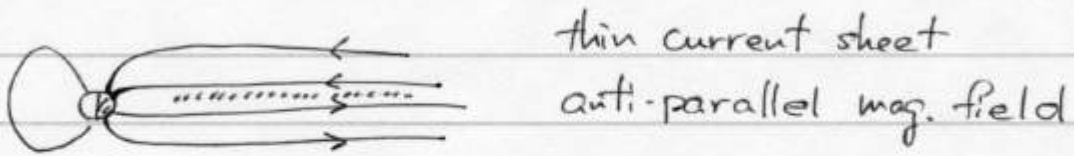
Substorm (reconnection model)

1. growth phase ($T \sim 40$ min.)

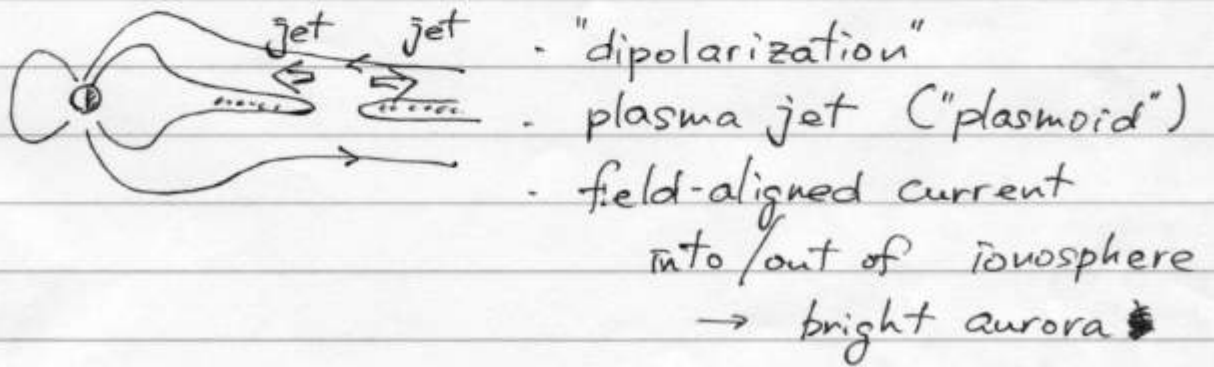


- magnetic field transport from dayside to tail (energy charge in tail)
- reconnection on dayside (southward IMF)

2. expansion phase ($T \sim 10$ min.)

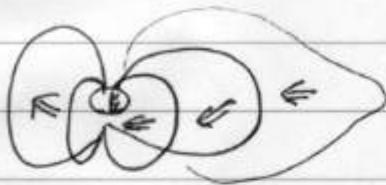


⇓ tail reconnection (energy discharge)



3. recovery phase ($T \sim$ hours)

- magnetic field transport back to dayside



ionospheric convection pattern (southward IMF)

