February 2009

Max Planck Institute for Solar System Research
Y. Narita, M. Fränz, N. Krupp with assistants
A. Angsmann, L. Guicking, K. Hallgren, and P. Kobel

Exercise for Sun-Planet Connections (2009) - Part 1

Space Weather

1.1 Space weather influence on spacecraft

Discuss what different effects space weather might have on spacecraft (and humans) in orbit around earth. Consider also the impact of different orbital parameters and if the spacecraft move to an interplanetary trajectory.

1.2 Parker spiral

(a) The solar wind plasma carries magnetic field lines with it, which attain a spiral shape due to the solar rotation as they extend through the solar system. This is the so-called *Parker spiral*.

Derive an equation which describes the flow of this plasma, assuming that it emerges from the solar surface ($r = r_0$) at the equator in radial direction (at the speed v) from a point at an arbitrary solar longitude ϕ_0. Write down two time-dependent equations in polar coordinates which describe the outward movement of this plasma in r and ϕ direction. Then combine these formulas to find a time-independent description for r, depending on v, ϕ, ϕ_0, r_0 and the angular velocity Ω of the solar rotation.

The resulting equation for r is a so-called *Archimedean spiral* (general form: $r = a+b\phi$ with real numbers a, b) which has the property that the distance between successive turnings of the spiral is always constant.

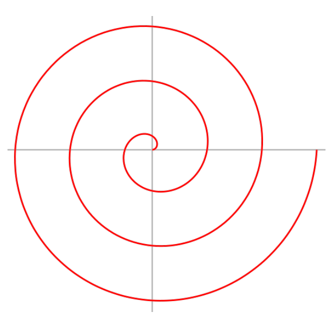

Figure 1: One arm of an Archimedean spiral.

(b) The aim of this exercise is to derive an equation in polar coordinates which relates the components B_r and B_ϕ to each other, depending on the radial distance r from the Sun, the radial velocity v_r of the solar wind and the Sun's angular velocity Ω. This will show us how far the magnetic field lines twist as they move outward with the plasma. For simplicity, stay within the equatorial plane, thus $B_\theta = v_\theta = 0$.

First, show that the fact that magnetic fields are source-free leads to the equation

$$B_r = B_0 \cdot \frac{r_0^2}{r^2} \tag{1}$$

where B_0 represents the magnetic field at r_0 (boundary condition). Next, assume a steady flow ($\vec{B}$ does not change with time) and use the generalized Ohm's law as well as Faraday's law to prove that

$$r\,(v_\phi B_r - v_r B_\phi) = \text{const.} \tag{2}$$

With $v_{\phi_0} = r_0\Omega$ and the assumption that the initial magnetic field emerging from the sun is purely radial, derive an equation for B_ϕ, depending only on B_r, v_r, v_ϕ, r and Ω (start from equation (2) and use equation (1) to eliminate B_0 and r_0). You can then further simplify the resulting equation for large distances from the Sun, where $r\Omega \gg v_\phi$.

Hints:

- *Divergence of a vector field $\vec{F}$ in spherical coordinates:*
$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(F_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi} \tag{3}$$
- *Curl of a vector field $\vec{F}$ in spherical coordinates:*
$$\begin{aligned}\vec{\nabla} \times \vec{F} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(F_\phi \sin\theta\right) - \frac{\partial F_\theta}{\partial\phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial}{\partial r}\left(rF_\phi\right)\right)\hat{\theta}\\ &\quad + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rF_\theta\right) - \frac{\partial F_r}{\partial\theta}\right)\hat{\phi}\end{aligned}$$
- *Generalized Ohm's law: $\vec{j} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$, where $\sigma \to \infty$ in this case, as the conductivity of plasmas is very high*
- *Faraday's law: $\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E}$*
- *we are regarding an axially symmetric system in $\hat{\phi}$, thus $\frac{\partial \vec{v}}{\partial \phi} = \frac{\partial \vec{B}}{\partial \phi} = 0$*

(c) Using the equation derived in part (b) and assuming a constant radial solar wind particle velocity of $\approx$ 400 km/s, calculate the angle of the interplanetary magnetic field lines (compared to the radial direction) at the orbit of the Earth (1 AU = $1.5 \cdot 10^{11}$ m). The sidereal rotation period of the Sun at the equator is 25.05 days.

Which value does this angle approach for very large distances from the Sun?

1.3 Electric power input

How large is the electric power input from the solar wind to the Earth magnetosphere (see figure below)? Use the values of $v = 400\,[\text{km/s}]$ (solar wind speed), $B = 5\,[\text{nT}]$ (interplanetary magnetic field), and the radius $R = 10\,\text{R}_\text{E}$ for the magnetosphere cross section. Hint: The Poynting flux $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ [W/m^2] gives an estimate of electric power. $R_E = 6400$ km, $\mu_0 = \frac{1}{4\pi} \cdot 10^{-4}\frac{\text{H}}{\text{m}}$

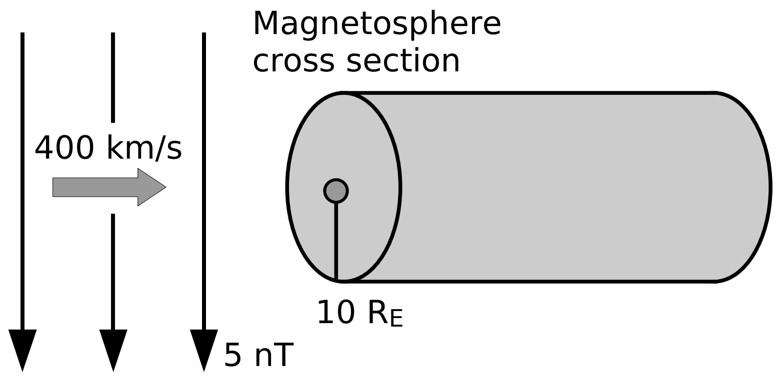

Figure 2: Cylinder magnetosphere in a plasma flow.

1.4 Induced voltage

1994 an space tether experiment was flown on the Space Shuttle. Estimate the induced voltage potential on a tether attached to the space shuttle. The experiment in question took place at an altitude of 300 km and with an orbital inclination of 28°.